

Mathematics

Teacher Guide



**California High School
Exit Examination**



California High School Exit Examination Mathematics Teacher Guide

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Introduction

The California High School Exit Examination (CAHSEE) Teacher Guide for mathematics is designed to provide comprehensive and accessible information to assist teachers in preparing students for the CAHSEE. Teachers are encouraged to reproduce sections or all of the guide for classroom use. Districts and school personnel are encouraged to use this material in staff development activities.

- “Purpose and Content” provides an overview of the CAHSEE.
- “Overview of the Strands” gives detailed information about how the California academic content standards for mathematics are tested on the CAHSEE. The following mathematics standards are assessed on the CAHSEE:

Number Sense

Statistics, Data Analysis, and Probability

Algebra and Functions

Measurement and Geometry

Mathematical Reasoning

Algebra I

The overview provides a summary of the essential knowledge and skills covered, followed by information about how the standard may be tested. For each standard, a released test question from a previous administration of the CAHSEE is provided with an explanation of the correct answer and an analysis of the incorrect answers.

Purpose and Content

The primary purpose of the CAHSEE is to significantly improve pupil achievement in public high schools and to ensure that pupils who graduate from public high schools can demonstrate grade-level competency in reading, writing, and mathematics. The CAHSEE helps identify students who are not developing skills that are essential for life after high school and encourages districts to give these students the attention and resources necessary to help them achieve these skills during their high school years.

The CAHSEE assesses a range of difficulty levels consistent with good testing practices. Questions assess full mastery of the designated academic content standards as well as foundational knowledge and skills underlying these standards, as recommended by the High School Exit Examination Standards Panel.

In mathematics, standards from grades six and seven and Algebra I are included on the CAHSEE because these academic content standards represent both foundational and competency standards that students should meet to graduate from high school.

All questions on the examination have been evaluated for their appropriateness for measuring the designated mathematics academic content standards. They have been reviewed and approved by committees of California educators, including teachers, administrators, and academicians. All items have also been reviewed and approved by California educators for their adherence to the principles of fairness and have been evaluated to determine if bias exists with respect to characteristics, such as gender, ethnicity, and language.

The test blueprints for the CAHSEE, which indicate the academic content standards tested and the number of items per standard, are available on the CAHSEE Web site.

The CAHSEE is not a timed test, which means it has no fixed time limit in which students must complete the examination. However, students are expected to complete their work during the regular school day unless their Individualized Educational Program (IEP) or Section 504 Plan specifies the need for extra time beyond the school day.

Mathematics Academic Content Standards

As mentioned above, the standards for the mathematics part of the CAHSEE are taken from the California academic content standards in grades six and seven and in Algebra I. They include standards from the following mathematical strands: Number Sense; Statistics, Data Analysis, and Probability; Algebra and Functions; Measurement and Geometry; Mathematical Reasoning; Algebra I.

The mathematics part of the CAHSEE contains 92 multiple-choice test questions (80 operational questions and 12 field-test questions).

Overview of the Strands

The Mathematics part of the CAHSEE assesses designated California academic content standards from grades six and seven and Algebra I. A multiple-choice format is used to assess six strands: Number Sense; Statistics, Data Analysis, and Probability; Algebra and Functions; Measurement and Geometry; Mathematical Reasoning; and Algebra I. Each of these strands is described in detail in the following section of the Teacher Guide. For reporting purposes, the Statistics, Data Analysis, and Probability strands for grades six and seven are combined. The Mathematical Reasoning questions, which are always based on concepts in Number Sense; Statistics, Data Analysis, and Probability; Algebra and Functions; and Measurement and Geometry, are reported under those strands. Thus, there is no reporting category specifically for Mathematical Reasoning.

The CAHSEE focuses on mathematics constructs that are taught and assessed throughout elementary, middle, and high school.

Although questions for the mathematics part of the CAHSEE do not specifically test students on mathematics vocabulary, they may require students to understand mathematical terms. It is especially important that students know the terms that appear in the language of the academic content standards associated with a question.

The following pages of the Teacher Guide discuss the mathematics strands and academic content standards included in the CAHSEE. The mathematics strands are:

- Number Sense
- Statistics, Data Analysis, and Probability
- Algebra and Functions
- Measurement and Geometry
- Mathematical Reasoning
- Algebra I

After each strand is described, each standard in that strand is discussed in detail, and a sample released test question is provided to illustrate each standard. Also included are possible reasons that students might select the distractors, as well as an explanation of the correct answer.

Teachers will find this section of the guide useful in understanding how the California academic content standards are assessed on the CAHSEE. A thorough understanding of the standards and the test questions associated with them will help teachers focus their instruction on the academic content standards and better prepare students for the exam.



Number Sense

14 questions

12 questions

17 questions

17 questions

8 questions

12 questions

**Number
Sense****Statistics,
Data
Analysis,
Probability****Algebra
and
Functions****Measurement
and
Geometry****Math
Reasoning****Algebra I**

CAHSEE contains 14 Number Sense items. Students' understanding of fractions, decimals, percents, and integers and their relationship to each other and to the other disciplines of mathematics is an essential component of their mathematics learning. CAHSEE questions in the Number Sense strand require students to demonstrate a foundational understanding of numbers and ways they are represented.

Students will be asked to:

- solve problems with fractions, decimals, and percents.
- compare and order numbers.
- demonstrate an understanding of percents, including those less than 1 and greater than 100.
- understand and meaningfully interpret large and small numbers in scientific notation.
- use specific characteristics of numbers, such as multiples, factors, and primes.
- use and represent integers as the basis for the comparison of quantities.

Essential to success in this CAHSEE strand is the student's understanding of the mathematical operations and the ways they are related to each other. This understanding includes:

- the meaning of arithmetic operations with fractions, decimals, and integers.
- the associative and commutative properties of addition and multiplication.
- the distributive property of multiplication over addition.
- the understanding and use of inverse relationships of addition and subtraction, multiplication, and division.
- finding square roots, squaring numbers, and using the inverse relationship between them.

Students also should possess computational fluency. They should be able to select appropriate methods and tools for computing with fractions and decimals; perform mental arithmetic; use algorithms for computing with fractions, decimals, and integers; use strategies for estimation and for judging the reasonableness of results; and be able to analyze and explain methods for solving problems with proportions.

The ten California academic content standards covered by the CAHSEE Number Sense strand are discussed in the following pages.

Strand	Number Sense (NS)	<p>The radius of the earth's orbit is 150,000,000,000 meters. What is this number in scientific notation?</p> <p>A 1.5×10^{-11}</p> <p>B 1.5×10^{11}</p> <p>C 15×10^{10}</p> <p>D 150×10^9</p> <p>M00213</p>
Standard	7NS1.1	

Read, write, and compare rational numbers in scientific notation (positive and negative powers of 10) with approximate numbers using scientific notation.

1 test question

Scientific notation is required knowledge in science and engineering because many numbers are either so large or so close to zero that there is no other convenient way to write them. CAHSEE questions in this standard require students to demonstrate understanding of the basic concepts of scientific notation using approximations of very large and very small numbers. Test questions may also involve the translation of approximate numbers into scientific notation, the comparison of numbers in scientific notation with either positive or negative exponents, and the understanding of the relative size of two numbers in scientific notation.

Sample Test Question

The correct answer is choice B. Students should recognize that the place-value distance from the 1 (highest place value, 100 billion) to the decimal is 11 digits and that this value provides an appropriate representation of the equivalence as 10^{11} , also equivalent to $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$. Students also should know that expressions in scientific notation may include a multiplier, between 1 and 10, along with an exponential value of 10. While exact powers of 10 are expressed without a multiplier (e.g., 10^{11}), numbers such as 150,000,000,000 require a multiplier along with the equivalent power of 10. A typical method of finding the multiplier and the exponent is to count the number of decimal places the decimal must move to create a number between 1 and 10. In the example, the decimal point is moved 11 places to the left to get 1.5 for the multiplier and $+11$ for the exponent.

Analysis of Distractors

Distractor A: expressed the power of 10 as $\frac{1}{10^{11}}$ (students are often unsure of the direction the decimal point should move)

B: correct answer

Distractor C: equivalent in number to 150,000,000,000, but not in scientific notation

Distractor D: equivalent in number to 150,000,000,000, but not in scientific notation

Strand **Number Sense (NS)**

Standard **7NS1.2**

Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers.

3 test questions

John uses $\frac{2}{3}$ of a cup of oats per serving to make oatmeal. How many cups of oats does he need to make 6 servings?

A $2\frac{2}{3}$

B 4

C $5\frac{1}{3}$

D 9

M23015

All students should understand the basic arithmetic functions involving rational numbers in all forms and be comfortable performing calculations with positive and negative numbers. CAHSEE questions in this standard require students to demonstrate computational fluency with rational numbers and an understanding of the relationships between these types of numbers. These skills are fundamental to achievement of the California academic content standards in mathematics.

Sample Test Question

The correct answer is choice B. Students should understand that multiplying the initial amount of oats by the 6 servings yields the easiest calculation.

Analysis of Distractors

Distractor A: added the two numbers and made a mistake in the calculation $\left(\frac{2}{3} + \frac{6}{1} = \frac{8}{3} = 2\frac{2}{3}\right)$

B: correct answer

Distractor C: subtracted $\frac{2}{3}$ from the 6 servings

Distractor D: divided 6 by $\frac{2}{3}$

Strand	Number Sense (NS)	<p>Some students attend school 180 of the 365 days in a year. About what part of the year do they attend school?</p> <p>A 18%</p> <p>B 50%</p> <p>C 75%</p> <p>D 180%</p> <p>M00047</p>
Standard	7NS1.3	
<p>Convert fractions to decimals and percents and use these representations in estimations, computations and applications.</p>		
<p>2 test questions</p>		

Students using mathematics in their daily lives will need to know how to convert decimals to fractions to percents with ease. The recognition of equivalent forms is essential for student fluency with numbers. CAHSEE questions in this standard require students to demonstrate the ability to find equivalent values and representations for numbers as well as to find and compute values using fractions, decimals, and percents. Students must also be able to use these various representations for estimating and performing computations in mathematical applications.

Sample Test Question

The correct answer is choice B. Students should recognize that the correct value is found by dividing the part (180) by the whole (365), giving $180 \div 365 \approx 0.493$. The question also requires conversion of 0.493 to a percent by multiplying by 100 ($0.493 = 49.3\%$). Finally, students should recognize that the phrase “about what part” in the stem calls for a rounded percent rather than an exact value and that $49.3\% \approx 50\%$. It is to be expected that many students will use mental estimation skills, rather than computation, to determine that 180 is about half of 365.

Analysis of Distractors

Distractor A: converted 180 to 18%

B: correct answer

Distractor C: incorrect computation of the decimal equivalent of $\frac{180}{365}$ or a possible guess with apparent plausibility

Distractor D: converted 180 to 180%

Strand **Number Sense (NS)**

Standard **7NS1.6**

Calculate the percentage of increases and decreases of a quantity.

1 test question

The cost of an afternoon movie ticket last year was \$4.00. This year an afternoon movie ticket costs \$5.00. What is the percent increase of the ticket from last year to this year?

A 10%

B 20%

C 25%

D 40%

M02158

CAHSEE questions in this standard require students to demonstrate understanding of percent increase and decrease, which is a fundamental tool in analyzing numerical information. For example, a price change of one dollar can be very meaningful in terms of buying a loaf of bread and inconsequential in terms of buying a car. Students should understand that percent change clarifies the impact of this kind of change, and they should be able to calculate the change easily. Standard 7NS1.6 will also be assessed with questions that require students to find the percent decrease. Finding the quantity that results from a given percentage increase or decrease is assessed in standard 7NS1.7.

Sample Test Question

The correct answer is choice C. Students should understand that finding the percent increase or decrease of a quantity requires first finding the difference between the initial value and the final value. In the sample question, the difference is \$1.00. Then to find the percent increase, students

must know to compare the difference to the initial cost by using division: $\frac{\$1.00}{\$4.00}$. Finally, the

resulting decimal must be converted to its equivalent percent: $1 \div 4 = 0.25 = 25\%$.

Analysis of Distractors

Distractor A: took the difference between the two ticket prices and incorrectly converted it to a percentage

Distractor B: multiplied both given values and converted to a percentage

C: correct answer

Distractor D: converted the original ticket price to a percentage

Strand	Number Sense (NS)	Sally puts \$200.00 in a bank account. Each year the account earns 8% simple interest. How much interest will be earned in three years? A \$16.00 B \$24.00 C \$48.00 D \$160.00
Standard	7NS1.7	
Solve problems that involve discounts, markups, commissions, and profit, and compute simple and compound interest.		
2 test questions		

M02119

CAHSEE questions in this standard require students to solve a variety of problems involving percents. Both consumers and people working in business need to understand the mathematical meaning of common business terms such as commission, tips, profit, and loss, as well as how to make interest and tax computations. Solving problems of these types is one of the most important skills students need as they become adults. Understanding these concepts and their applications can assist students in managing their money and other resources. This standard will also be assessed with test questions that require students to find simple and compound interest, as well as discounts, markups, taxes, tips, and commissions. A maximum of three iterations is used for questions that involve calculating compound interest. The iterations include the initial multiplication of principal by interest rate.

Sample Test Question

The correct answer is choice C. Students should recognize that simple interest is calculated by multiplying the principal by the annual rate and then multiplying by the time. In the sample question, the principal is \$200, the rate is 8%, and the time is 3 years. To calculate correctly, students are also required to convert 8% to its decimal equivalent ($\$200 \cdot 0.08 \cdot 3 = \48).

Analysis of Distractors

Distractor A: one year interest; multiplied the principal and interest rate only

Distractor B: multiplied the rate times the number of years only

C: correct answer

Distractor D: converted the interest rate to a decimal incorrectly ($\$200 \cdot 0.8$), and did not multiply by the number of years

Strand **Number Sense (NS)**Standard **7NS2.1****Understand negative whole-number exponents. Multiply and divide expressions involving exponents with a common base.****1 test question**

$$\frac{10^{-2}}{10^{-4}} =$$

A 10^{-6}

B 10^{-2}

C 10^2

D 10^8

M02832

CAHSEE questions in this standard require students to understand the concept of negative exponents. One of the most powerful concepts in mathematics is that exponential notation can be extended to include new concepts. Negative exponents are an example of this kind of extension.

Sample Test Question

The correct answer is choice C. Students should understand that 10^{-2} is equivalent to

$\frac{1}{10^2}$ and $\frac{1}{10^{-4}}$ is equivalent to 10^4 . Thus, it is possible to represent the problem as

$\frac{10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10}$, making the underlying concept more apparent. $\frac{10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10}$ can be reduced

by dividing it by $\frac{10 \cdot 10}{10 \cdot 10}$ to $10 \cdot 10$, which is equivalent to 10^2 . Once the students master the

concept behind negative exponents, they understand that when dividing exponential expressions

with the same base, the exponents must be subtracted, so that $\frac{10^{-2}}{10^{-4}}$ is equivalent to $10^{(-2)-(-4)}$,

which is equivalent to 10^2 .

Analysis of Distractors

Distractor A: added the exponents or made an error in the subtraction of $(-4) - (-2)$

Distractor B: did not apply the negative signs correctly in the same subtraction

C: correct answer

Distractor D: multiplied -2 by -4

Strand **Number Sense (NS)**

Standard **7NS2.2**

Add and subtract fractions by using factoring to find common denominators.

1 test question

Which fraction is equivalent to $\frac{5}{6} + \frac{7}{8}$?

A $\frac{35}{48}$

B $\frac{6}{7}$

C $\frac{20}{21}$

D $\frac{41}{24}$

M12713

The focus of this content standard is on the students' ability to add and subtract fractions with unlike denominators that share one or more factors. Students should be able to find the prime factorization of each denominator, then combine factors to determine the least common denominator. CAHSEE questions in this standard require students to perform addition and subtraction using equivalent fractions with common denominators. The algorithmic approach of this standard is associated with the requirement that common denominators be determined by factoring.

Sample Test Question

The sample requires students to find the needed common denominator for 6 and 8 using prime factors. The correct answer choice is D. Students should recognize that the prime factors for 6 are 2 and 3 and that the prime factors for 8 are 2 and 4. Since the common prime factor is 2, including the additional factors of 3 and 4 gives $2 \cdot 3 \cdot 4$ as the prime factors of the least common denominator. Students should then find equivalent fractions using the least common denominator and add the fractions.

Analysis of Distractors

Distractor A: multiplied the numerators and denominators together, respectively

Distractor B: added the numerators and denominators together, respectively and reduced the fraction to its lowest form

Distractor C: the numerators of the equivalent fractions expressed as a numerator and denominator

D: correct answer

Strand **Number Sense (NS)**

Standard **7NS2.3**

Multiply, divide, and simplify rational numbers by using exponent rules.

1 test question

$$(3^8)^2 =$$

A 3^4

B 3^6

C 3^{10}

D 3^{16}

M02406

CAHSEE questions in this standard require students to select the appropriate rules for operations with exponents with common bases and perform accurate computations in simplifying rational numbers. Students should understand the following rules:

- adding exponents when multiplying numbers with common bases
- subtracting exponents when dividing numbers with common bases
- multiplying exponents when raising a number to a particular power

Questions may include those requiring multistep operations, such as the simplification of numerators and denominators with common factors.

Sample Test Question

The correct answer is choice D. In this instance, students should use the rule for multiplying exponents $\left[(a^b)^c = a^{bc}\right]$ to determine that $(3^8)^2 = 3^{16}$.

Analysis of Distractors

Distractor A: used division instead of multiplication for the exponents

Distractor B: used subtraction instead of multiplication for the exponents

Distractor C: used addition instead of multiplication for the exponents

D: correct answer

Strand	Number Sense (NS)	The square root of 150 is between—
Standard	7NS2.4	A 10 and 11.
Use the inverse relationship between raising to a power and extracting the root of a perfect square integer; for an integer that is not square, determine without a calculator the two integers between which its square root lies and explain why.		B 11 and 12.
		C 12 and 13.
		D 13 and 14.
		M02666

1 test question

CAHSEE questions in this standard require students to demonstrate a conceptual understanding of powers and roots and their inverse relationship. The idea of mathematical inverse is a key precursor for algebraic reasoning, and students should understand that taking a root is the inverse operation of raising a number or expression to a power. For example, students should know that $\sqrt{9^2} = 9$. Students will not be required to calculate the square root for a number that is not a perfect square, but students should be able to approximate the value of the square root of an integer that is not a perfect square.

Sample Test Question

The correct answer is choice C. Students should recognize that they must first determine both the closest perfect square greater than the given integer and the closest perfect square less than the integer. This task may involve some trial-and-error multiplication along with the application of knowledge of squares and square roots. Since the number 150 is not a perfect square, students may recall or calculate that 144 is a perfect square and that 169 is the next perfect square. Since 144 is close to but less than 150 and 169 is close to but greater than 150, the square root of 150 must lie between those two perfect squares.

Analysis of Distractors

The distractors misplace 150 between other numbers and may be selected by students who do not understand the concept of square root or who may incorrectly calculate the square of one of the numbers.

Strand	Number Sense (NS)	<p>If $x = 3$, what is the value of x?</p> <p>A -3 or 0</p> <p>B -3 or 3</p> <p>C 0 or 3</p> <p>D -9 or 9</p> <p>M02122</p>
Standard	7NS2.5	

Understand the meaning of the absolute value of a number; interpret the absolute value as the distance of the number from zero on a number line; and determine the absolute value of real numbers.

1 test question

CAHSEE questions in this standard require students to demonstrate a conceptual understanding of absolute value and its meaning as represented on a number line. Relating absolute value to distance on the number line may help students understand the concept: Distance cannot be negative, but there will always be two numbers on the number line that are the same distance from zero. Questions may require students to find the absolute value after performing a basic computation.

Sample Test Question

The correct answer is choice B. Students should recognize that since the absolute value of a number is the distance on a number line from that number to zero in either direction, all absolute values are positive numbers: $|x| = x$ and $|-x| = x$. In the test question, if $|x| = 3$, then $x = -3$ or $x = 3$, since $|3| = 3$ and $|-3| = 3$.

Analysis of Distractors

Distractor A: equated the absolute value of 3 with 0 and a negative value of 3

B: correct answer

Distractor C: incorrectly equated the absolute value of 3 with 0 and a value of 3

Distractor D: associated the absolute value of 3 with the square of 3



Statistics, Data Analysis, and Probability

14 questions

12 questions

17 questions

17 questions

8 questions

12 questions

Number
Sense
**Statistics,
Data
Analysis,
Probability**
Algebra
and
FunctionsMeasurement
and
GeometryMath
Reasoning

Algebra I

CAHSEE contains 12 Statistics, Data Analysis, and Probability items. To demonstrate knowledge and skills in the Statistics, Data Analysis, and Probability strand, students must understand the fundamental concepts involved in data collection, display, and analysis. Students will be asked to determine ways to collect, organize, and display relevant data to answer questions; formulate questions that can be addressed with data; select and use appropriate statistical methods to analyze data; and develop and evaluate inferences and predictions that are based on data. In addition, students are required to understand and apply the basic concepts of probability.

Specifically, the standards in the Statistics, Data Analysis, and Probability strand include the following knowledge and skills:

- finding measures of central tendency to characterize data
- interpreting and evaluating conclusions based on data
- organizing and representing possible outcomes for events and expressing theoretical probabilities
- representing probabilities as ratios, proportions, and percents
- understanding the numerical continuum of probability between impossibility (0) and certainty (1)
- recognizing the difference between independent and dependent events
- displaying data appropriately, including both one- and two-variable data sets

The seven California academic content standards covered by the CAHSEE Statistics, Data Analysis, and Probability strand are discussed in the following pages.

Strand	Statistics, Data Analysis, and Probability (PS)	Rico's first three test scores in biology were 65, 90, and 73. What was his mean score?
Standard	6PS1.1	A 65
	Compute the range mean, median, and mode of data sets.*	B 73
		C 76
		D 90

M02247

One of the major objectives of the Statistics, Data Analysis, and Probability strand is to give students tools to help them understand the uses and misuses of statistics. This CAHSEE content standard has three components: computation of the mean, computation of the median, and recognition of the mode of data sets. Statistical measures of central tendency represent important methods for summarizing and comparing single-variable data sets. Students should understand the significance of each as a measure of central tendency as well as the differences among these measures. For this standard, students will not be asked to find the median of an even number of values.

Students should know that:

- the median is the middle score of an ordered set of numbers, where half the scores are greater than the median and half are less.
- the mode is the number which appears most frequently.
- the mean is most affected by extreme values.

Sample Test Question

The correct answer is choice C. Students should recognize that they should compute the mean by first finding the sum ($65 + 90 + 73 = 228$) and then dividing by 3 ($228 \div 3 = 76$).

Analysis of Distractors

Distractor A: minimum of the data set

Distractor B: median of the data set

C: correct answer

Distractor D: maximum of the data set

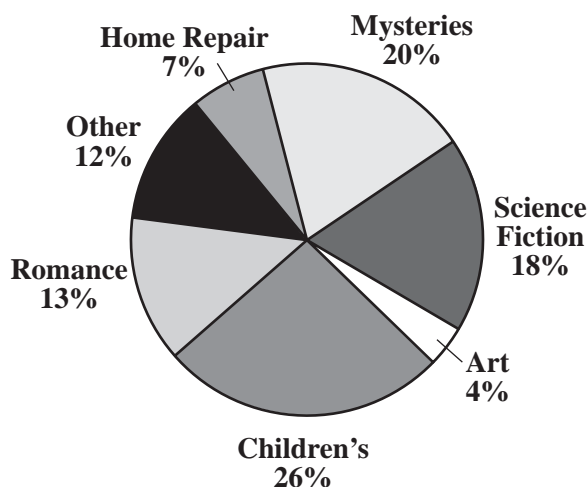
* The CAHSEE test blueprint does not include the crossed-out portion of this content standard.

Strand **Statistics,
Data Analysis, and
Probability (PS)**

Standard **6PS2.5**
**Identify claims based on statistical
data and, in simple cases, evaluate
the validity of the claims.**

1 test question

**The Smithburg town library wanted to see what
types of books were borrowed most often.**



According to the circle graph shown above—

- A** more Children's books were borrowed than Romance and Science Fiction combined.
- B** more than half of the books borrowed were Children's, Mysteries, and Art combined.
- C** more Mysteries were borrowed than Art and Science Fiction combined.
- D** more than half of the books borrowed were Romance, Mysteries, and Science Fiction combined.

M02131

This content standard has two components: identifying claims made on the basis of statistical data and evaluating the validity of the claims based on statistical data. Because students should be able to understand statistical claims as well as they understand purely verbal arguments for or against a position, students should develop skills to evaluate the quality of data and conclusions based on data. CAHSEE questions for this standard may ask students to identify a valid claim based on data or to recognize a question for which the data could be used to provide an answer.

Sample Test Question

The correct answer is D. Students should be able to add the percentages from Romance, Mysteries, and Science Fiction and get 51%, thus making option D the correct one.

Analysis of Distractors

Distractor A: The percentage of Children's books (26) is not more than the sum of the percentages of Romance and Science Fiction ($13 + 18$).

Distractor B: The sum of the percentages of Children's, Mysteries, and Art ($26 + 20 + 4$) is exactly 50, which is not more than half.

Distractor C: The percentage of Mysteries (20) is not more than the sum of the percentages of Art and Science Fiction ($4 + 18$).

D: correct answer

Strand **Statistics,
Data Analysis, and
Probability (PS)**

Standard **6PS3.1**
**Represent all possible outcomes for
compound events in an organized way
(e.g., tables, grids, tree diagrams) and
express the theoretical probability of
each outcome.**

1 test question

To get home from work, Curtis must get on one of the three highways that leaves the city. He then has a choice of four different roads that lead to his house. In the diagram below, each letter represents a highway, and each number represents a road.

		Highway		
		A	B	C
Road	1	A 1	B 1	C 1
	2	A 2	B 2	C 2
	3	A 3	B 3	C 3
	4	A 4	B 4	C 4

If Curtis randomly chooses a route to travel home, what is the probability that he will travel Highway B and Road 4?

A $\frac{1}{16}$

B $\frac{1}{12}$

C $\frac{1}{4}$

D $\frac{1}{3}$

M02512

Organizing structures, such as sample spaces, diagrams, and tables, are useful for the representations of probabilities, and the ability to create a structured representation of a complex situation is an important reasoning tool. To demonstrate achievement in this standard, students must recognize appropriate and correct representations of events. From the correct representation, they must derive an understanding of the relationship between the frequency of the outcome and its numerical expression. They should be able to determine a theoretical probability of any particular outcome based on a correct representation.

Sample Test Question

The correct answer is choice B. Students should recognize that if there are n possible outcomes for an independent event and r possible outcomes for another independent event, there are nr outcomes for the two events together. In other words, if there are n ways to do one thing and r ways to do another thing, there are nr ways to do the two things together. Thus, three highways times four roads equals 12 routes, and the probability of selecting any individual route is $\frac{1}{12}$.

Analysis of Distractors

Distractor A: squared the probability of taking one road

B: correct answer

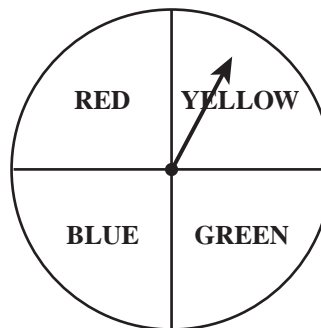
Distractor C: the probability of taking any one road

Distractor D: the probability of taking any one highway

Strand **Statistics,
Data Analysis, and
Probability (PS)**

Standard **6PS3.3**
**Represent probabilities as ratios,
proportions, decimals between
0 and 1, and percentages between
0 and 100 and verify that the
probabilities computed are
reasonable; know that if P is the
probability of an event, $1-P$ is the
probability of an event not
occurring.**

2 test questions



The spinner shown above is fair. What is the probability that the spinner will NOT stop on red if you spin it one time?

- A $\frac{1}{4}$
- B $\frac{1}{3}$
- C $\frac{3}{4}$
- D $\frac{4}{3}$

M00094

All students should understand that mathematical probability is used to predict what might happen in the future and that probabilities are ratios determined by considering the likely results or outcomes of events. CAHSEE questions for this standard cover all of the components of the standard. Students are expected to:

- know that probabilities are ratios that can be expressed as fractions, decimals, or percentages.
- compute the probability of a described event.
- verify the reasonableness of a computed probability.
- compute the probability that an event will not occur.

Sample Test Question

The correct answer is choice C. Students should recognize that because the four regions covered by the spinner have equal areas, the probability of the spinner stopping on any one region is $\frac{1}{4}$.

Therefore, the probability that it will **not** stop on a given region is $1 - \frac{1}{4}$ or $\frac{3}{4}$.

Analysis of Distractors

Distractor A: the probability that the event will occur (this value was not subtracted from 1)

Distractor B: misconception—only three quadrants were used to calculate the probability; the probability of the spinner landing on one of the three quadrants

C: correct answer

Distractor D: used a fraction to represent the total number of quadrants divided by three quadrants (all but red) or obtained the correct answer but then inverted it

Strand	Statistics, Data Analysis, and Probability (PS)	<p>A bag contained four green balls, three red balls, and two purple balls. Jason removed one purple ball from the bag and did NOT put the ball back in the bag. He then randomly removed another ball from the bag. What is the probability that the second ball Jason removed was purple?</p> <p>A $\frac{1}{36}$</p> <p>B $\frac{1}{9}$</p> <p>C $\frac{1}{8}$</p> <p>D $\frac{2}{9}$</p> <p style="text-align: right;">M03097</p>
Standard	6PS3.5	
<p>Understand the difference between independent and dependent events.</p> <p>1 test question</p>		

Discerning the difference between dependent and independent events is important in evaluating probabilistic outcomes. CAHSEE questions in this content standard require students to understand that events are independent of each other if the occurrence or non-occurrence of one event does not affect the probability of the occurrence or non-occurrence of another event. Similarly, students must recognize that events are dependent if the occurrence or non-occurrence of one event affects the probability of the occurrence or non-occurrence of another event. Computation may be required to determine the result of the independent or dependent events.

Sample Test Question

The correct answer is choice C. Students should determine that initially the bag contains 4 green, 3 red, and 2 purple balls, for a total of 9 balls. When 1 purple ball is removed and not replaced, the bag contains a total of 8 balls. Since there are now 8 balls with only 1 being purple, the probability of randomly choosing the purple ball is $\frac{1}{8}$.

Analysis of Distractors

Distractor A: the probability of randomly choosing both purple balls

Distractor B: the probability of randomly choosing 1 purple ball from 9 balls

C: correct answer

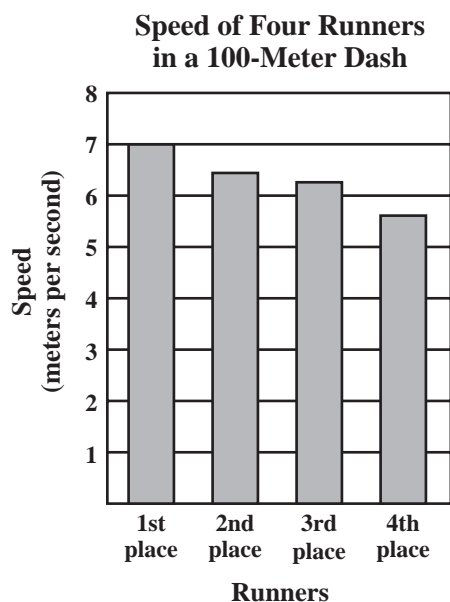
Distractor D: added the two probabilities for an independent event $\left(\frac{1}{9} + \frac{1}{9}\right)$

Strand **Statistics,
Data Analysis, and
Probability (PS)**

Standard **7PS1.1**

Know various forms of display for data sets, including a stem-and-leaf plot or box-and-whisker plot; use the forms to display a single set of data or to compare two sets of data.*

2 test questions



Based on the bar graph shown above, which of the following conclusions is true?

- A** Everyone ran faster than 6 meters per second.
- B** The best possible rate for the 100-meter dash is 5 meters per second.
- C** The first-place runner was four times as fast as the fourth-place runner.
- D** The second-place and third-place runners were closest in time to one another.

M00279

Large data sets are difficult to grasp mentally without an accessible visual representation. CAHSEE questions in this standard require students to recognize and interpret various forms of display and to compare two sets of data displayed the same way. The forms of display for single-variable data sets assessed on the CAHSEE include bar graphs, line graphs, scatterplots, pictographs, circle graphs, and Venn diagrams. CAHSEE questions for this standard may also require students to select an appropriate type of data display. Items for this standard may also focus on the students' ability to extract and manipulate data from a data display for use in a computation.

* The CAHSEE test blueprint does not include the crossed-out portion of this content standard.

Sample Test Question

The correct answer is choice D. Students should use the graph to determine the speed of each runner and then evaluate and compare the four answer choices. From the graph, the first-place runner's speed was approximately 7 meters per second (m/s); the second-place runner's speed was approximately 6.5 m/s; the third-place runner's speed was approximately 6.3 m/s; and the fourth-place runner's speed was approximately 5.6 m/s. This data display shows that the second-place and third-place runners' times were closest together.

Analysis of Distractors

Distractor A: incorrect because the fourth-place runner ran at approximately 5.6 m/s

Distractor B: incorrect because all four runners ran faster than 5 m/s

Distractor C: incorrect because 7 m/s is not four times faster than 5.6 m/s

D: correct answer

Strand **Statistics, Data Analysis, and Probability (PS)**

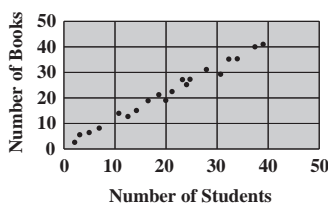
Standard **7PS1.2**

Represent two numerical variables on a scatterplot and informally describe how data points are distributed and any apparent relationship that exists between the two variables (e.g., between time spent on homework and grade level).

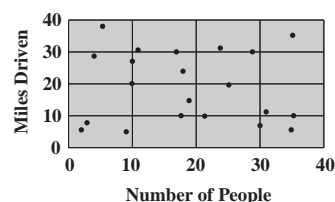
2 test questions

Which scatterplot shows a negative correlation?

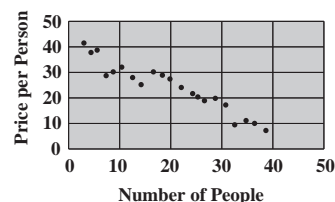
A



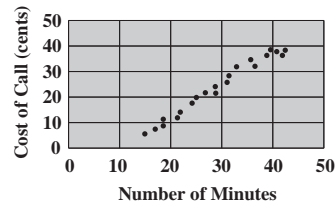
C



B



D



M02546

The identification of patterns and relationships, including clustering and trends, as well as the concept of correlation (positive, negative, or none) are significant aspects of using data. Students should understand correlation as a measure of the relationship between two variables, with negative correlation as the association of an increase in the value of one variable with a decrease in the corresponding value of the second variable. Students should also recognize that positive correlation is the association of an increase in the value of one variable with an increase in the corresponding value of the second variable.

CAHSEE questions for this standard address the following components of the standard:

- representing two variables on a scatterplot
- determining the distribution of the variables
- recognizing the general relationships between the two variables represented.

Sample Test Question

The correct answer is choice B: As the price per person increases, the number of people decreases, indicating a negative correlation.

Analysis of Distractors

Distractor A: represents a positive correlation

B: correct answer

Distractor C: represents no correlation

Distractor D: represents a positive correlation



Algebra and Functions

14 questions

12 questions

17 questions

17 questions

8 questions

12 questions

**Number
Sense**
**Statistics,
Data
Analysis,
Probability**
**Algebra
and
Functions**
**Measurement
and
Geometry**
**Math
Reasoning**
Algebra I

CAHSEE contains 17 Algebra and Functions items. The Algebra and Functions strand is most closely associated with the representation of quantitative relationships, such as functions, equations, graphs, geometric diagrams, and verbal expressions. As students increase their mathematical knowledge and skills, they work frequently with algebraic symbols, expressions with variables, and graphical representations. It is essential that students develop an understanding of several different meanings and uses of variables through multiple representations. Everyday experiences with linear functions should aid in the development of the concepts of proportionality and the ability to discriminate between linear and nonlinear functions. Students must also learn to recognize and generate equivalent expressions, solve linear equations, and effectively use formulas.

To demonstrate achievement in this strand, students will be asked to:

- work with patterns and relationships.
- represent, analyze, and generalize a variety of patterns with tables, graphs, and symbolic rules.
- compare different forms of representations.
- identify functions.
- use algebraic expressions.
- solve linear equations.

The use of mathematical models to represent and understand quantitative relationships is developed by modeling and solving contextualized problems. The analysis of change in various contexts involves tools such as graphs to analyze the nature of changes in quantities in linear relationships.

The ten California academic content standards covered by the CAHSEE Algebra and Functions strand are discussed in the following pages.

Strand	Algebra and Functions (AF)	Which of the following inequalities represents the statement, “A number, x, decreased by 13 is less than or equal to 39”?
Standard	7AF1.1	
Use variables and appropriate operations to write an expression, an equation, an inequality, or a system of equations or inequalities that represents a verbal description (e.g., three less than a number, half as large as area A).		<p>A $13 - x \geq 39$</p> <p>B $13 - x \leq 39$</p> <p>C $x - 13 \leq 39$</p> <p>D $x - 13 < 39$</p>
2 test questions		M03049

Translating verbal descriptions into mathematical expressions is essential in solving real-world problems. CAHSEE questions in this standard require students to translate between verbal descriptions and mathematical equivalents. Students should be able to use variables and appropriate operations to write or identify an expression, an equation, a system of equations or inequality to solve a problem.

Sample Test Question

The correct answer is choice C. Students should recognize that “a number, x , decreased by 13” is represented as $x - 13$ and that “less than or equal to 39” is represented by ≤ 39 . Putting both parts of the statement together, $x - 13 \leq 39$.

Analysis of Distractors

Distractor A: represents “13 decreased by a number, x , and also greater than or equal to 39”

Distractor B: used the correct inequality notation but, like Distractor A, represents “13 decreased by a number, x ”

C: correct answer

Distractor D: represents the appropriate expression for “a number, x , decreased by 13” but represents “less than 39,” rather than “less than or equal to 39”

Strand	Algebra and Functions (AF)	<div>If $h = 3$ and $k = 4$, then $\frac{hk + 4}{2} - 2 =$</div> <div>A 6</div> <div>B 7</div> <div>C 8</div> <div>D 10</div> <div>M00052</div>
Standard	7AF1.2	
Use the correct order of operations to evaluate algebraic expressions such as $3(2x + 5)^2$.		
1 test question		

CAHSEE questions for this standard require students to select and use the correct order of arithmetic operations in evaluating expressions (parentheses, exponents, multiplication, division, addition, subtraction). Students may also be required to evaluate expressions that include the distributive property and other basic properties of real numbers.

Sample Test Question

The correct answer is choice A. Students should first substitute 3 and 4 for h and k , then multiply h by k , add 4, divide by 2, and then subtract 2:

$$\frac{(3)(4) + 4}{2} - 2 = \frac{12 + 4}{2} - 2 = \frac{16}{2} - 2 = 8 - 2 = 6.$$

Analysis of Distractors

A: correct answer

Distractor B: divided by 2 before evaluating the numerator

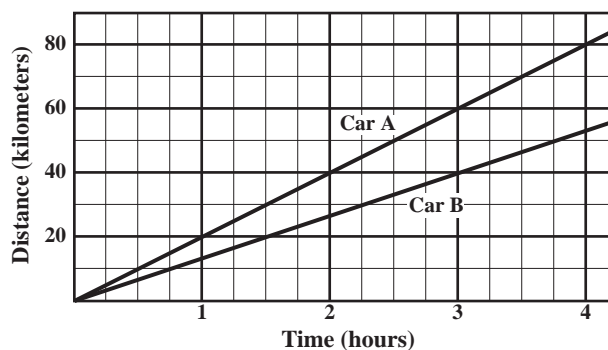
Distractor C: divided the product of 3 and 4 by 2 before evaluating the numerator

Distractor D: added 4 before multiplying 3 by 4

Strand **Algebra and Functions (AF)**

Standard **7AF1.5**
Represent quantitative relationships graphically and interpret the meaning of a specific part of a graph in the situation represented by the graph.

3 test questions



After three hours of travel, Car A is about how many kilometers ahead of Car B?

- A** 2
- B** 10
- C** 20
- D** 25

M00066

CAHSEE questions for this standard focus on either of its two main components. The first component requires the selection and execution of a graph that accurately and appropriately represents a quantitative relationship. The second component requires the interpretation of information presented in graphical form. All graphs in items from this standard must represent an algebraic relationship.

Sample Test Question

The correct answer is choice C. Students should recognize that distance, in kilometers, is recorded on the y -axis, and time, in hours, is recorded on the x -axis. Three hours on the time scale corresponds to 60 kilometers for Car A, and three hours corresponds to 40 kilometers for Car B. The number of kilometers that Car A is ahead of Car B after 3 hours is represented by the difference between the distance traveled by Car A and the distance traveled by Car B in the same time (60 kilometers – 40 kilometers = 20 kilometers).

Analysis of Distractors

Distractor A: the number of hours that Car A traveled when it had gone 40 kilometers; the distance Car B traveled in 3 hours

Distractor B: made an error when reading the scale of the graph, assumed that the increments had a value of 10

C: correct answer

Distractor D: used the approximate difference in distance at 4 hours, rather than 3

Strand	Algebra and Functions (AF)	$x^3y^3 =$
Standard	7AF2.1	A $9xy$
Interpret positive whole-number powers as repeated multiplication and negative whole-number powers as repeated division or multiplication by the multiplicative inverse. Simplify and evaluate expressions that include exponents.		B $(xy)^6$
		C $3xy$
		D $xxxyyy$
1 test question		M02879

This standard has four main components: the concept of positive whole-number powers as repeated multiplication, the concept of negative whole-number powers as repeated division, multiplication by the multiplicative inverse, and simplification and evaluation of expressions that include exponents. The first two components of this standard are assessed in the related Number Sense standards 2.1 and 2.3. CAHSEE questions that assess student achievement in this standard may require students to evaluate monomial expressions. Other questions for this standard may require students to demonstrate an understanding of the multiplicative inverse.

Sample Test Question

The correct answer is choice D. Students should recognize that $x^3 = x \cdot x \cdot x$ and that $y^3 = y \cdot y \cdot y$, so that $x^3y^3 = x \cdot x \cdot x \cdot y \cdot y \cdot y = xxxyyy$.

Analysis of Distractors

Distractor A: multiplied the exponents and used the product as a coefficient

Distractor B: added the exponents

Distractor C: moved the value of the exponents to serve as a coefficient

D: correct answer

Strand	Algebra and Functions (AF)	Simplify the expression shown below.
Standard	7AF2.2	$(6a^4bc)(7ab^3c)$
Multiply and divide monomials; extend the process of taking powers and extracting roots to monomials when the latter results in a monomial with an integer exponent.		A $13a^4b^3c$
1 test question		B $13a^5b^4c^2$
		C $42a^4b^3c$
		D $42a^5b^4c^2$
		M02109

CAHSEE questions in this standard require students to multiply and divide monomials, expand powers, and find roots for monomials when the results are integer exponents. Items may also include combinations of multiplying and dividing monomials. Students should be comfortable with the rules for multiplying and dividing exponential expressions with the same base.

Sample Test Question

The correct answer is choice D, as $6 \cdot 7 = 42$ and the product of a^4 and a is a^5 ; the product of b and b^3 is b^4 ; and the product of c and c is c^2 .

Analysis of Distractors

Distractor A: added 6 and 7 instead of multiplying and failed to use a as a^1 , b as b^1 , and c as c^1 when adding the values of the exponents

Distractor B: added 6 and 7 instead of multiplying

Distractor C: multiplied the exponents incorrectly, as in Distractor A

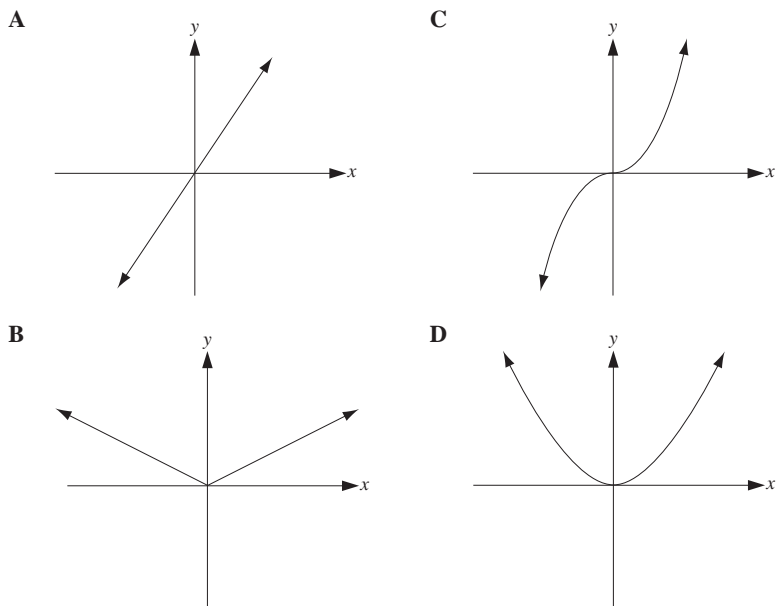
D: correct answer

Strand **Algebra and Functions (AF)**

Standard **7AF3.1**
Graph functions of the form $y = nx^2$ and $y = nx^3$ and use in solving problems.

1 test question

Which of the following could be the graph of $y = x^3$?



M02200

CAHSEE questions in this standard require knowledge of graphing basic quadratic functions and cubic functions, as demonstrated by selecting the appropriate graph of a given function or by selecting the appropriate function for a given graph. Some questions may also require knowledge of function graphing to solve problems. As part of their foundational understanding of functions, students should be able to predict the shape of a graph based on the characteristics of the given function (e.g., linear, quadratic).

Sample Test Question

The correct answer is choice C. Students should understand the basic concepts underlying the problem—that cubic functions are nonlinear and that negative values for x correspond to negative values for y and positive values for x correspond to positive values for y .

Analysis of Distractors

Distractor A: a linear graph, rather than a nonlinear function

Distractor B: the graph of an absolute value function

C: correct answer

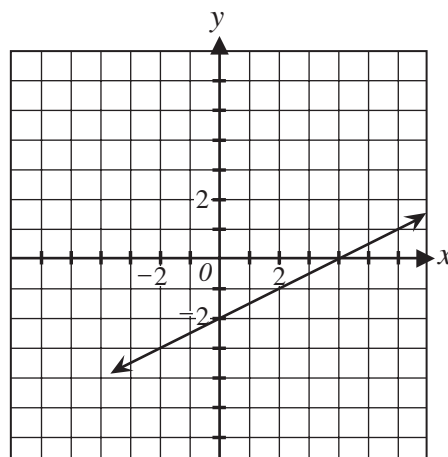
Distractor D: the graph of a quadratic function

Strand **Algebra and Functions (AF)**

Standard **7AF3.3**

Graph linear functions, noting that the vertical change (change in y -value) per unit of horizontal change (change in the x -value) is always the same and know that the ratio (“rise over run”) is called the slope of a graph.

2 test questions



What is the slope of the line shown in the graph above?

A -2

B $-\frac{1}{2}$

C $\frac{1}{2}$

D 2

M02556

Students should understand that linear functions can model many real-world phenomena and that the rate of change in a function is shown by the slope of the graph of the function. A conceptual understanding of slope can be a key element in students' development of proportional reasoning skills. CAHSEE questions for this standard may focus on either of its two main components. The first component is graphing linear functions on the xy -coordinate system. The second is the identification of the slope in quantitative terms from a given linear function or the selection of a given slope from a numerical value, from a line shown on a graph, or from two pairs of coordinate points.

Sample Test Question

The correct answer is choice C. Students should understand slope as the change in y divided by the change in x and/or as the ratio “rise over run.” In this problem, the change in the y -value is obtained by subtracting -2 from 0 , and the change in the x -value is obtained by subtracting

0 from 4 , and thus $\frac{\text{Change in } y}{\text{Change in } x} = \frac{0 - (-2)}{4 - 0} = \frac{2}{4} = \frac{1}{2}$.

Analysis of Distractors

Distractor A: the value of the y-intercept, misunderstood the concept of slope

Distractor B: made an error in the subtraction of $0 - (-2)$ or $4 - 0$

C: correct answer

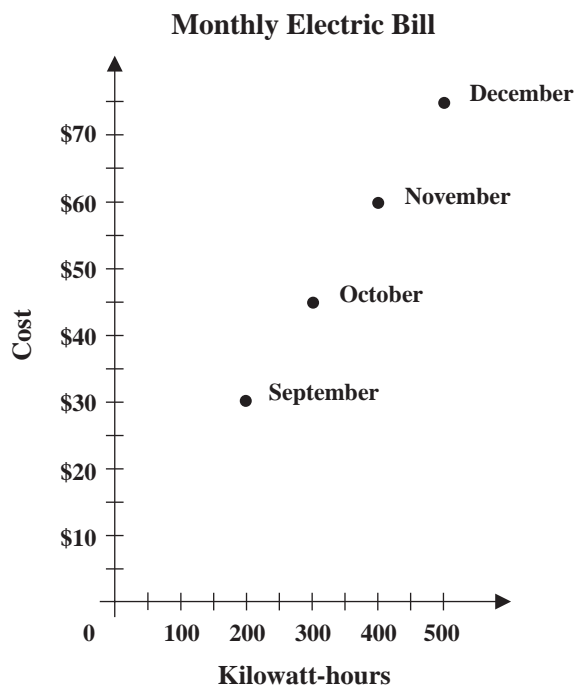
Distractor D: divided the change in x by the change in y

Strand **Algebra Functions (AF)**Standard **7AF3.4**

Plot the values of quantities whose ratios are always the same (e.g., cost to the number of an item, feet to inches, circumference to diameter of a circle). Fit a line to the plot and understand that the slope of a line equals the quantities.

1 test question

The graph below shows Francine's electric bill for 4 different months. What is the price per kilowatt-hour of Francine's electricity?



- A** \$0.15
- B** \$0.30
- C** \$1.50
- D** \$6.67

M02681

Graphing direct variation is a powerful way to comprehend and express proportional reasoning. CAHSEE questions within this standard focus on either of its two main components, both of which require students to understand the relationship between the graphical presentation of data and the symbolic representation of data. The first component involves the identification of the correct graph. The second component involves the determination of the slope of a direct variation and the interpretation of the meaning of the slope as a constant ratio between the two quantities in the variation.

Sample Test Question

The correct answer is choice A. Students must understand the relationship between the kilowatt-hours on the x -axis and the corresponding cost on the y -axis. After studying the given data points, the students should recognize that the relationship is linear. The students should then divide the y -value by the x -value for any of the given points (September, October, November, or December). The result will be a constant \$0.15.

Analysis of Distractors

A: correct answer

Distractor B: used the incorrect x -value in the calculation ($\$30 \div 100 = \0.30)

Distractor C: made a decimal error in the calculation

Distractor D: reversed the correct order by dividing the x -value by the y -value
($200 \div \$30 = \6.67)

Strand	Algebra and Functions (AF)	In the inequality $2x + \\$10,000 \geq \\$70,000$, x represents the salary of an employee in a school district. Which phrase most accurately describes the employee's salary?
Standard	7AF4.1	
Solve two-step linear equations and inequalities in one variable over the rational numbers, interpret the solution or solutions in the context from which they arose, and verify the reasonableness of the results.		A At least \$30,000
		B At most \$30,000
		C Less than \$30,000
3 test questions		D More than \$30,000

M02621

CAHSEE questions for this standard may focus on any of its components, including solving two-step linear equations, solving two-step inequalities, interpreting the solutions of equations or inequalities, and judging the reasonableness of the solutions of equations or inequalities.

Sample Test Question

The correct answer is choice A. Students should recognize that the inequality has the solution $x \geq \$30,000$ and that the correct way to state this inequality is “at least \$30,000.”

Analysis of Distractors

The distractors offer incorrect solutions for the inequality and/or ways to express the mathematical notation.

Strand	Algebra and Functions (AF)	Stephanie is reading a 456-page book. During the past 7 days she has read 168 pages. If she continues reading at the same rate, how many more days will it take her to complete the book? A 12 B 14 C 19 D 24 <div style="text-align: right;">M00380</div>
Standard	7AF4.2	
	Solve multistep problems involving rate, average speed, distance, and time or a direct variation.	
	2 test questions	

Problem solving is a significant higher-order thinking skill that enables students to apply their mathematical knowledge to real-world situations. CAHSEE questions for this standard may require students either to solve a specific multistep problem or to determine the equation that should be used to solve the problem. Questions may also require students to understand the concept of direct variation and to recognize that direct variation may also be expressed as a linear function. This standard is closely related to Algebra I standard 15.0, which requires students to solve a variety of problems such as rate, work, and percent mixture using algebraic methods.

Sample Test Question

The correct answer choice is A. One method is to subtract 168 from 456 to obtain the number of pages left. Then set up the proportion $\frac{168}{7} = \frac{288}{x}$ and solve for x .

Analysis of Distractors

A: correct answer

Distractor B: the number of pages read per day, assuming 168 pages is the amount of pages left,

can be found by $\frac{168}{12}$

Distractor C: the total days that are needed to read the book can be found from the proportion

$$\frac{168}{7} = \frac{456}{x}$$

Distractor D: the number of pages read per day can be found by $\frac{456}{19}$



Measurement and Geometry

14 questions

12 questions

17 questions

17 questions

8 questions

12 questions

**Number
Sense**
**Statistics,
Data
Analysis,
Probability**
**Algebra
and
Functions**
**Measurement
and
Geometry**
**Math
Reasoning**
Algebra I

CAHSEE contains 17 Measurement and Geometry items. As students relate their experiences from earlier classroom instruction in measurement and geometry to situations in their everyday lives, their knowledge and ability to apply this knowledge increase in depth and sophistication. To demonstrate understanding in this CAHSEE strand, students must be able to select and use appropriate units; estimate and calculate measurements for the length, area, and volume of geometric figures; understand scaling in scale drawings and how changes in linear dimension affect area and volume; and solve problems involving dimensional analysis and conversion from one unit to another.

To demonstrate achievement of the knowledge and skills in the measurement component of this strand, students should be able to use both metric and customary units of measurement for the following:

- determining the relationship between different units within the same system and converting from one unit to another within and between measurement systems
- using scale drawings and models to determine measurements of the original
- solving problems involving dimensional analysis for rates and other compound units
- relating the effect of changing the choice of a linear unit on the related square and cubic units for area and volume, respectively

The geometry component of this strand includes computing the perimeter, area, and volume of the most common 2- and 3-dimensional figures and using these common figures to estimate or compute the area of more complex objects.

To demonstrate acquisition of the knowledge and skills in the geometry component of the strand, students should be able to:

- describe, classify, and understand relationships between length, area, and volume among types of 2- and 3-dimensional objects.
- use coordinate geometry to represent and examine the properties of figures and their images under translation and reflection.
- understand and use the Pythagorean theorem.
- recognize and demonstrate understanding of congruence in terms of the sides and angles of 2-dimensional figures.

When CAHSEE items require students to use formulas, the formulas are provided within parentheses in the stem. All formulas the students may use, including the estimated value of π , will be provided *except* for the following:

- perimeter of a polygon (sum of the sides)
- circumference of a circle ($C = 2\pi r$, $C = \pi d$)
- area of a triangle $\left(A = \frac{1}{2}bh\right)$
- area of a parallelogram (including rectangles and squares, $\text{base} \times \text{height}$)
- volume of a rectangular prism ($\text{length} \times \text{width} \times \text{height}$)

The formula for finding the area of a nontraditional figure, such as a rhombus, will be provided.

In estimation problems, the approximate value of π that should be used to obtain the correct answer will be provided as part of the stem. Otherwise, the answer will include π (e.g., $12 + 3\pi$).

The ten California academic content standards covered by the CAHSEE Measurement and Geometry strand are discussed in the following pages.

Strand	Measurement and Geometry (MG)	One millimeter is—
Standard	7MG1.1	A $\frac{1}{1000}$ of a meter.
Compare weights, capacities, geometric measures, times, and temperatures within and between measurement systems (e.g., miles per hour and feet per second, cubic inches to cubic centimeters).		B $\frac{1}{100}$ of a meter.
2 test questions		C 100 meters.
		D 1000 meters.
		M00276

Comparing units and computing the effect of changing units are essential skills for acquiring knowledge in mathematics and science disciplines. CAHSEE questions for this standard require students to convert between two units of measurement within the same system or between two different systems of measurement. Conversion formulas are provided for test questions that require the student to convert between less commonly used units (such as pints to gallons), systems of measurement, square units, and cubic units.

Sample Test Question

The correct answer is choice A. Students must know that to make this conversion, 1 millimeter is equal to $\frac{1}{1000}$ of a meter.

Analysis of Distractors

A: correct answer

Distractor B: used $\frac{1}{100}$ as the conversion factor

Distractor C: used 100 as the conversion factor

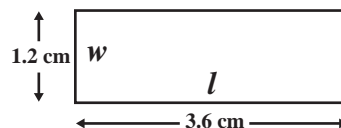
Distractor D: used 1000 as the conversion factor

Strand **Measurement and Geometry (MG)**

Standard **7MG1.2**
Construct and read drawings and models made to scale.

1 test question

The actual width (w) of a rectangle is 18 centimeters (cm). Use the scale drawing of the rectangle to find the actual length (l).



- A** 6 cm
- B** 24 cm
- C** 36 cm
- D** 54 cm

M02087

The ability to move back and forth between a scale drawing or model and a real object is essential for understanding representation as well as proportional reasoning. Questions on the CAHSEE focus on the second component of this standard, reading scale drawings and models. Students will be asked to read and interpret drawings and scale models. Students may also be asked to apply given measurements to determine the scale of a figure.

Sample Test Question

The correct answer is choice D. Students must recognize that the ratio between the width of the scale drawing and the actual width of the rectangle is the same as that between the length of the scale drawing and the actual length. Students may use a variety of approaches to solve the

problem, including setting up a proportion similar to $\frac{1.2}{18} = \frac{3.6}{l}$ and solving for l .

Analysis of Distractors

Distractor A: used an incorrect relationship between the parts of the proportion,
 $(18 \div 3.6) \cdot 1.2 = 6$

Distractor B: subtracted the width from the length and multiplied by 10

Distractor C: used $3.6 \cdot 10$ rather than $3.6 \cdot 15$

D: correct answer

Strand	Measurement and Geometry (MG)	Sixty miles per hour is the same rate as which of the following?
Standard	7MG1.3	<p>A 1 mile per minute</p> <p>B 1 mile per second</p> <p>C 6 miles per minute</p> <p>D 360 miles per second</p>
Use measures expressed as rates (e.g., speed, density) and measures expressed as products (e.g., person-days) to solve problems; check the units of the solutions; and use dimensional analysis to check the reasonableness of the answer.		M02473
2 test questions		

Dimensional analysis is a critical skill in physical science, engineering, and the social sciences. Students should be familiar with the rates named in the standard as well as other commonly used measures (e.g., kilowatt-hours, foot-pounds, acre-feet, person-hours). Questions on the CAHSEE for this standard focus on all three of its components: measures as rates, measures as products, and reasonableness of results.

Sample Test Question

The correct answer is choice A. Students may reason that another way to express the given rate is 60 miles per 60 minutes, since 1 hour is equivalent to 60 minutes. The expression is 60 miles/hour is equivalent to 60 miles/60 minutes and that since $60 \div 60 = 1$, the rate is equivalent to 1 mile/minute. Students should also be encouraged to evaluate the answer choices in the question for their reasonableness as a restatement of this real-world rate.

Analysis of Distractors

A: correct answer

Distractor B: equated 1 hour with 60 seconds

Distractor C: divided 60 miles per hour by 10, rather than by 60

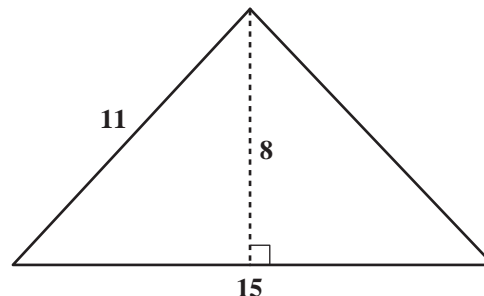
Distractor D: multiplied 60 by 6

Strand **Measurement and Geometry (MG)**

Standard **7MG2.1**

Use formulas routinely for finding the perimeter and area of basic two-dimensional figures and the surface area and volume of basic three-dimensional figures, including rectangles, parallelograms, trapezoids, squares, triangles, circles, prisms, and cylinders.

3 test questions



What is the area of the triangle shown above?

- A** 44 square units
- B** 60 square units
- C** 88 square units
- D** 120 square units

M00101

All students should know how to compute the area and volume of basic figures and also how to apply basic formulas in many areas to solve problems. This standard requires students to find the perimeter and area of 2-dimensional figures and the surface area and volume of 3-dimensional figures. The figures tested on the CAHSEE include parallelograms, trapezoids, triangles, circles, prisms, and cylinders. Additional items for this standard may ask students to find the length of a side of a figure when given the length of a known side, perimeter, area, or a combination of these values. Students are required to know the following formulas:

- perimeter of a polygon (sum of the sides)
- circumference of a circle ($C = 2\pi r$, $C = \pi d$)
- area of a triangle $\left(A = \frac{1}{2}bh\right)$
- area of a parallelogram (including rectangles and squares, $\text{base} \times \text{height}$)
- volume of a rectangular prism ($\text{length} \times \text{width} \times \text{height}$)

These formulas will not be given with the test questions. The estimated value of π will be given unless π is included in the answer choices.

Sample Test Question

The correct answer is choice B. Students should know the area formula $A = \frac{1}{2}bh$ and use it to determine that the height is 8 units and the base is 15 units. The area is 60 square units.

Analysis of Distractors

Distractor A: used 11 as the base, rather than 15

B: correct answer

Distractor C: used 11 as the base and did not take $\frac{1}{2}$ of the product of the base and the height

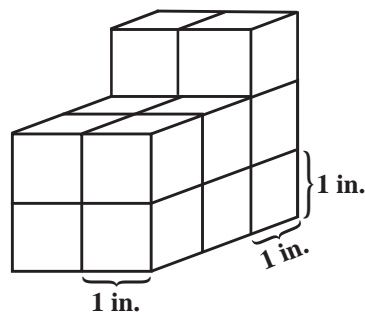
Distractor D: did not take $\frac{1}{2}$ of the product of the base and the height

Strand **Measurement and Geometry (MG)**

Standard **7MG2.2**
Estimate and compute the area of more complex or irregular two- and three-dimensional figures by breaking the figures down into more basic geometric objects.

2 test questions

One-inch cubes are stacked as shown in the drawing below.



What is the total surface area?

- A** 19 in.²
- B** 29 in.²
- C** 32 in.²
- D** 38 in.²

M02812

This standard emphasizes the development of problem-solving skills with visual tools. CAHSEE questions for this standard focus on both of its components. The first component requires students to identify the measurable and/or computable parts of a shape or structure. This identification task is largely dependent on the ability to visualize the familiar geometric structures that make up a more complex figure. Where estimation is required, students may be given dimensions of known shapes, scale and proportion, or grids in the visual prompt. The second component requires students to estimate and/or compute the area of the subdivided portions of a figure. Frequently, the dimensions of the component parts are not given directly by labeled measurements but must be determined by such means as adding or subtracting lengths or extending lines.

Sample Test Question

The correct answer is choice D. The foreground portion of the object contains a 2 by 2 by 2-cube grouping, and the background contains a 1 by 2 by 3-cube grouping. To find the surface area, students must visually assemble the surface of the individual cubes that make up the entire object and recognize that the object has 8 planar faces: right, rear, left, front, bottom, front top, rear face, and rear top. The surface areas of each corresponding face are 7, 6, 7, 4, 6, 4, 2, and 2 square inches, and the sum of the areas of these faces is 38 square inches.

Analysis of Distractors

Distractor A: included only half of the exterior faces

Distractor B: did not include the left or right face and either the rear face or rear top

Distractor C: did not include the bottom or rear of the figure

D: correct answer

Strand

Measurement and
Geometry (MG)

Standard

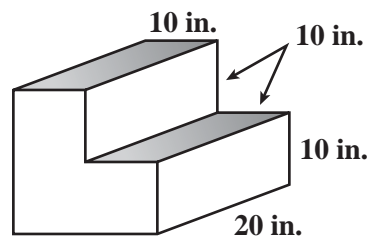
7MG2.3

Compute the length of the perimeter, the surface area of the faces, and the volume of a three-dimensional object built from rectangular solids.

Understand that when the lengths of all dimensions are multiplied by a scale factor, the surface area is multiplied by the square of the scale factor and volume is multiplied by the cube of the scale factor.

1 test question

The short stairway shown below is made of solid concrete. The height and width of each step is 10 inches (in.). The length is 20 inches.



What is the volume, in cubic inches, of the concrete used to create this stairway?

- A 3000
- B 4000
- C 6000
- D 8000

M02990

This standard contains two components. The first component focuses on the students' ability to compute the length of the perimeter, the surface area of the faces, and the volume of a three-dimensional object built from rectangular solids.

The second component focuses on the students' understanding of the results of multiplying the lengths of all dimensions by a scale factor, i.e., the surface area is multiplied by the square of the scale factor or the volume is multiplied by the cube of the scale factor. Items addressing the scale factor component of this standard may include two-dimensional objects.

Sample Test Question

The correct answer is choice C. Students should find the volume of the larger rectangular prism by multiplying 10 times 20 times 20. They should then find the volume of the smaller rectangular prism by multiplying 10 times 10 times 20. The sum of these 2 volumes gives the correct answer.


Analysis of Distractors

Distractor A: found the volume of the larger prism by multiplying 10 times 10 times 20 and the volume of the smaller prism by multiplying 10 times 10 times 10

Distractor B: found the volume of only the larger prism

C: correct answer

Distractor D: doubled the volume of the larger prism

<p>Strand Measurement and Geometry (MG)</p> <p>Standard 7MG2.4 Relate the changes in measurement with a change of scale to the units used (e.g., square inches, cubic feet) and to conversion between units (1 square foot = 144 square inches or $[1 \text{ ft}^2] = [144 \text{ in.}^2]$, 1 cubic inch is approximately 16.38 cubic centimeters or $[1 \text{ in.}^3] = [16.38 \text{ cm}^3]$).</p> <p>1 test question</p>	<p>The width of the rectangle shown below is 6 inches (in.). The length is 2 feet (ft).</p> <div data-bbox="898 296 1304 422">  <p style="text-align: right; margin-right: 50px;">6 in.</p> <p style="text-align: center;">2 ft</p> </div> <p>What is the area of the rectangle in square inches?</p> <p>A 12</p> <p>B 16</p> <p>C 60</p> <p>D 144</p> <p style="text-align: right;">M03243</p>
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Unit conversions for area and volume have practical applications in students' lives. CAHSEE questions for this standard require students to relate the changes in measurement with a change of scale to the units used and to convert between units. The emphasis is on the relationship between linear units for distance, square units for area, and cubic units for volume. Conversion formulas will be provided for linear measures except for inches to feet to yards or conversions within the metric system.

Sample Test Question

The correct answer choice is D. Students should convert 2 feet to 24 inches and multiply 6 and 24 to get the area of the rectangle.

Analysis of Distractors

Distractor A: multiplied 6 and 2

Distractor B: found the perimeter of the rectangle using the given values without making the conversion of feet to inches

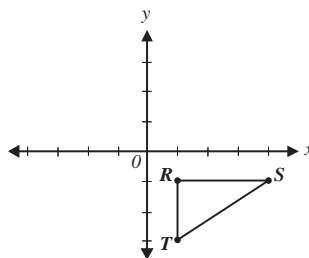
Distractor C: made the conversion of feet to inches, but then found the perimeter of the rectangle

D: correct answer

Strand **Measurement and Geometry (MG)**

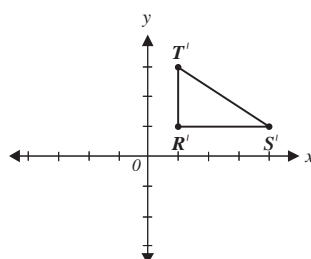
Standard **7MG3.2**
Understand and use coordinate graphs to plot simple figures, determine lengths and areas related to them, and determine their image under translations and reflections.

2 test questions

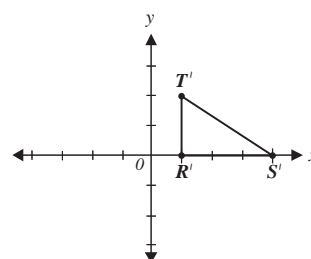


Which of the following triangles $R'S'T'$ is the image of triangle RST that results from reflecting triangle RST across the y -axis?

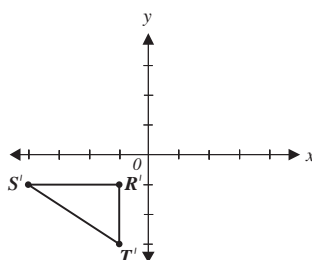
A



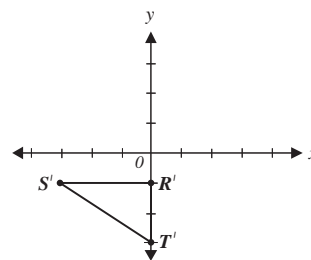
C



B



D



M02861

Understanding the xy coordinate system is an essential step in developing important thinking and problem-solving skills. Visualizing and manipulating objects are useful tools, not only in geometry but also in other disciplines.

CAHSEE questions for this standard assess students' understanding of all components of the standard: plotting simple figures with ordered pairs, determining lengths and areas from plotted figures, and finding images following transformations by translations and reflections.

Sample Test Question

The correct answer is choice B. Students must recognize the reflective correspondence between points R and R' , S and S' , and T and T' and that triangle $R'S'T'$ is the reflective image, across the y -axis, of triangle RST .

Analysis of Distractors

Distractor A: reflection of triangle *RST* across the *x*-axis

B: correct answer

Distractor C: reflection of triangle *RST* across the *x*-axis and its translation by 1 unit down or its

reflection across $y = -\frac{1}{2}$

Distractor D: reflection of triangle *RST* across the *y*-axis and its translation by 1 unit to the right

Strand

**Measurement and
Geometry (MG)**

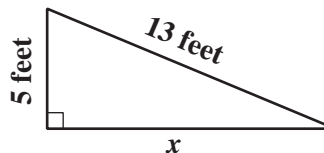
Standard

7MG3.3

Know and understand the Pythagorean theorem and its converse and use it to find the length of the missing side of a right triangle and the lengths of other line segments and, in some situations, empirically verify the Pythagorean theorem by direct measurement.

2 test questions

What is the value of x in the right triangle shown below?



- A** 8 feet
- B** 12 feet
- C** 18 feet
- D** 23 feet

M03181

The Pythagorean theorem is important for its problem-solving function as well as its role as a bridge between geometry and algebra. CAHSEE questions assessing this standard address the following components of the standard: using the Pythagorean theorem to find the length of the missing base, altitude, or hypotenuse of a right triangle and using the Pythagorean theorem to find lengths of line segments in figures other than triangles.

Sample Test Question

The correct answer is choice B. Students should use the Pythagorean theorem $5^2 + x^2 = 13^2$ to obtain the correct answer of 12.

Analysis of Distractors

Distractor A: the difference of the hypotenuse and altitude

B: correct answer

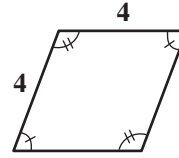
Distractor C: the sum of the hypotenuse and altitude

Distractor D: the sum of the hypotenuse and twice the altitude

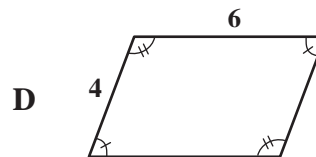
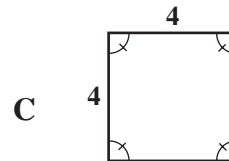
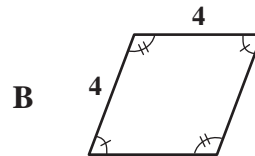
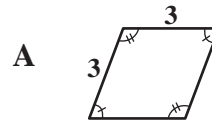
Strand **Measurement and Geometry (MG)**

Standard **7MG3.4**
Demonstrate an understanding of conditions that indicate two geometrical figures are congruent and what congruence means about the relationships between the sides and angles of the two figures.

1 test question



Which figure is congruent to the figure shown above?



M00020

CAHSEE questions for this standard require students to discriminate between figures that are or are not congruent to a given figure. Students may also be asked to identify specific reasons to support the assertion that two figures are congruent. Relationships between sides and angles of figures are significant because they may indicate the presence or absence of congruence by axiomatic reasoning, such as side-angle-side congruence. Items in this standard will not involve the use of acronyms for side and angle relationships of congruent figures (e.g., SAS, SSS).

Sample Test Question

The correct answer is choice B. Students should understand that two polygons are congruent if all corresponding sides and angles are congruent. Reasoning from this basis, students should recognize that the figure in choice B is congruent to the given figure because of the correspondence with sides of length 4 and angles with two marks. Students should also understand that because both figures are rhombi with non-congruent adjacent angles, all four sides are congruent and the opposite angles are congruent.

Analysis of Distractors

Distractor A: a similar, but not congruent figure

B: correct answer

Distractor C: a square

Distractor D: a parallelogram with non-congruent adjacent sides



Mathematical Reasoning

14 questions

12 questions

17 questions

17 questions

8 questions

12 questions

**Number
Sense**
**Statistics,
Data
Analysis,
Probability**
**Algebra
and
Functions**
**Measurement
and
Geometry**
**Math
Reasoning**
Algebra I

CAHSEE contains 8 Mathematical Reasoning items. Reasoning is an integral part of mathematics and requires several important skills, including examining patterns, making and testing conjectures, and using formal inductive and deductive reasoning to formulate mathematical arguments. Mastery of each of these elements of reasoning requires students to work with diverse problems and activities.

Activities in this strand require language with sufficient precision, clarity, and appropriateness to support rigorous thinking. Standards in mathematical reasoning require students to analyze problems by identifying relationships, to formulate and justify conjectures, to use estimation on the basis of numerical or graphical information, to use inductive and deductive reasoning, to evaluate the reasonableness of solutions, and to generalize results and apply them to new problems.

Each question in this strand is also classified within one of the four grades 6 and 7 mathematical strands for purposes of reporting student scores. None of the questions in this strand is classified in the Algebra I strand.

The six specific California academic content standards covered by the CAHSEE Mathematical Reasoning strand are discussed in the following pages.

Strand	Mathematical Reasoning (MR)	Chris drove 100 kilometers from San Francisco to Santa Cruz in 2 hours and 30 minutes. What computation will give Chris' average speed, in kilometers per hour?
Standard	7MR1.1 Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.	<p>A Divide 100 by 2.5.</p> <p>B Divide 100 by 2.3.</p> <p>C Multiply 100 by 2.5.</p> <p>D Multiply 100 by 2.3.</p>
2 test questions		M03164

Students must be able to analyze situations to clarify a problem and to identify those elements that will make it possible to solve the problem. CAHSEE questions for this standard emphasize the analysis of problems rather than their solutions. The components of the standard include the following: determining relationships, discriminating between relevant and irrelevant information, identifying missing information, sequencing and prioritizing information, and observing and identifying algebraic and geometric patterns.

Sample Test Question

The correct answer is choice A. Students must recognize that Chris' average speed is the unknown variable and must know how to use the distance, rate, and time equation to determine the rate. This question is classified in the Measurement and Geometry strand for purposes of reporting student scores.

Analysis of Distractors

A: correct answer

Distractor B: used division, but incorrectly used 2.3 as the value for 2 hours and 30 minutes

Distractor C: used multiplication, although the expression for the number of hours is correct

Distractor D: used multiplication and incorrectly used 2.3 as the value for 2 hours and 30 minutes

Strand	Mathematical Reasoning (MR)	If n is any odd number, which of the following is true about $n + 1$?
Standard	7MR1.2	<p>A It is an odd number.</p> <p>B It is an even number.</p> <p>C It is a prime number.</p> <p>D It is the same number as $n - 1$.</p>
Formulate and justify mathematical conjectures based on a general description of the mathematical question or problem posed.		M00155
1 test question		

The challenge for many students in formulating mathematical conjectures is precision of language. CAHSEE questions for this standard focus on both of its components: formulation of a conjecture and justification of a conjecture. Students may be asked to make conjectures based on indirect or incomplete evidence. Questions may state a conjecture and ask students to choose among reasons that the conjecture is reasonable.

Sample Test Question

The correct answer is choice B. Students should reason that if n is odd, $n + 1$ is necessarily even. Students may also attempt to find counterexamples, i.e., examples of odd n and also odd $n + 1$. This question is classified in the Algebra and Functions strand for purposes of reporting student scores.

Analysis of Distractors

Distractor A: always false because $n + 1$ must always be even

B: correct answer

Distractor C: true only when $n = 1$, since 2 is the only even prime

Distractor D: never true since $n - 1$ always differs from $n + 1$ by 2

Strand

Mathematical Reasoning (MR)

Standard

7MR2.1**Use estimation to verify the reasonableness of calculated results.****2 test questions****The table below shows the number of visitors to a natural history museum during a 4-day period.**

Day	Number of Visitors
Friday	597
Saturday	1115
Sunday	1346
Monday	365

Which expression would give the BEST estimate of the total number of visitors during this period?

- A** $500 + 1100 + 1300 + 300$
- B** $600 + 1100 + 1300 + 300$
- C** $600 + 1100 + 1300 + 400$
- D** $600 + 1100 + 1400 + 400$

M11112

This standard emphasizes estimating and evaluating the reasonableness of results. This standard requires students to use estimating skills in computation and compare estimated results to calculated results in order to judge their reasonableness.

Sample Test Question

The correct answer choice is C. Students should round each value to the nearest hundred. This question is classified in the Number Sense strand for purposes of reporting student scores.

Analysis of Distractors

Distractor A: rounded 597 to 500 and 365 to 300

Distractor B: rounded 365 to 300

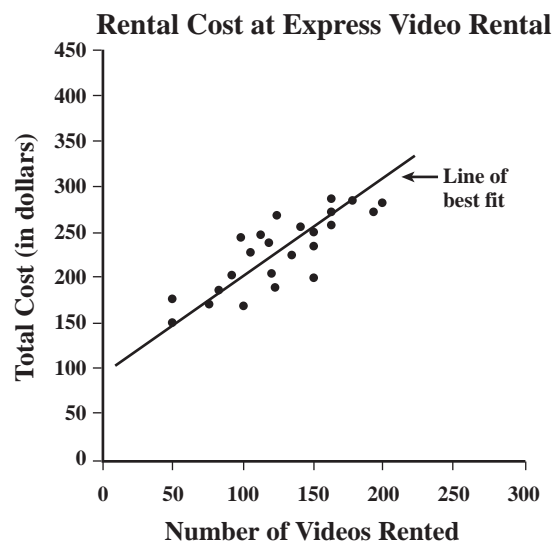
C: correct answer

Distractor D: rounded 1346 to 1400

Strand **Mathematical Reasoning (MR)**

Standard **7MR2.3**
Estimate unknown quantities graphically and solve for them by using logical reasoning and arithmetic and algebraic techniques.

1 test question



Using the line of best fit shown on the scatterplot above, which of the following best approximates the rental cost per video to rent 300 videos?

- A** \$3.00
- B** \$2.50
- C** \$2.00
- D** \$1.50

M02209

Graphs provide a quick summary of data or of a relationship but may not include the specific information required to answer a particular question. By identifying trends and patterns and using interpolation and extrapolation, students may be able to obtain a reasonable estimate of the needed information. This content standard has two components: estimating graphically and solving for unknown quantities. CAHSEE questions for the standard may ask students to find or identify the most accurate line of best fit through a scatterplot, to extract information from a graph by interpolation or extrapolation, or to identify an equation that could be used to solve a problem shown in a graph.

Sample Test Question

The correct answer is choice D. Students should recognize that extending the line to include an x -value that corresponds to 300 videos allows the interpretation of a corresponding value of the total cost on the y -axis. From the line of best fit, renting 300 videos corresponds to approximately \$450 or \$1.50 per video. This question is classified in the Statistics, Data Analysis, and Probability strand for purposes of reporting student scores.

Analysis of Distractors

Distractor A: a total cost of approximately \$900 to obtain a per video cost of \$3.00, which is out of the range of the line of best fit

Distractor B: a total cost of approximately \$750 to obtain a per video cost of \$2.50, also out of range of the line of best fit

Distractor C: a total cost of approximately \$600 to obtain a per video cost of \$2.00, also out of range of the line of best fit

D: correct answer

Strand

Mathematical Reasoning (MR)

Standard

7MR2.4

Make and test conjectures by using both inductive and deductive reasoning.

1 test question

The table below shows values for x and corresponding values for y .

x	y
21	3
14	2
28	4
7	1

Which of the following represents the relationship between x and y ?

A $y = \frac{1}{7}x$

B $y = 7x$

C $y = x - 6$

D $y = x - 18$

M00377

Being able to identify patterns (inductive reasoning) and then testing the validity of the patterns (deductive reasoning) are key skills in many fields besides mathematics. This standard requires students to use inductive and deductive reasoning to make and test conjectures. CAHSEE questions for this standard may require reasoning from general to specific, from specific to general, and by use of counterexample.

Sample Test Question

The correct answer is choice A. Students should be able to recognize that the values for y are $\frac{1}{7}$ that of the values for x . This question is classified in the Algebra and Functions strand for purposes of reporting student scores.

Analysis of Distractors

A: correct answer

Distractor B: the values of x multiplied by 7

Distractor C: correct only when the value of x is 7

Distractor D: correct only when the value of x is 21

Strand **Mathematical Reasoning (MR)**

Standard **7MR3.3**
Develop generalizations of the results obtained and the strategies used and apply them to new problem situations.

1 test question

Len runs a mile in 8 minutes. At this rate how long will it take him to run a 26-mile marathon?

Which of the following problems can be solved using the same arithmetic operations that are used to solve the problem above?

- A** Len runs 26 miles in 220 minutes. How long does it take him to run each mile?
- B** A librarian has 356 books to place on 18 shelves. Each shelf will contain the same number of books. How many books can the librarian place on each shelf?
- C** A cracker box weighs 200 grams. What is the weight of 100 boxes?
- D** Each basket of strawberries weighs 60 grams. How many baskets can be filled from 500 grams of strawberries?

M00137

One key problem-solving skill is to recognize how a new problem is like a simpler or more familiar problem. This content standard requires students to understand the process by which problems are reasoned, analyzed, and solved. CAHSEE questions for this standard may include the selection of appropriate analogs to a given problem situation, and the relevance of the analogs as applied to the types of reasoning, patterns of operations, or logical extensions, rather than to context or other more superficial characteristics of the problem. Questions for this standard may or may not require numerical solutions.

Sample Test Question

The correct answer is choice C. Students should recognize the analog in using multiplication to determine the total weight of 100 cracker boxes, with 1 cracker box weighing 200 grams. This question is classified in the Number Sense strand for purposes of reporting student scores.

Analysis of Distractors

Distractor A: requires the calculation of time (in minutes) per one mile, rather than per 26 miles as stated in the question

Distractor B: necessary to calculate the number of books per shelf by dividing 356 by 18

C: correct answer

Distractor D: similar to Distractor B, since to find the number of 60-gram baskets in 500 grams, the appropriate step is to divide 500 by 60



Algebra I

14 questions	12 questions	17 questions	17 questions	8 questions	12 questions
Number Sense	Statistics, Data Analysis, Probability	Algebra and Functions	Measurement and Geometry	Math Reasoning	Algebra I

CAHSEE contains 12 Algebra I items. The Algebra I strand builds upon students' knowledge and skills developed from their experience with linear functions, tables, graphs, verbal rules, and symbolic rules. As students deepen their understanding of relations and functions, they will expand their capacity to make meaningful use of new types of functions, including polynomial and exponential functions. Important new learning includes combining functions, expressing functions in equivalent forms, and finding inverses. This experience leads to more global understanding of classes of functions as a concept and the recognition of the significant characteristics of various classes.

To demonstrate achievement in the Algebra I strand, students must also develop insights into mathematical abstraction and structure. Students should develop an understanding of the algebraic properties that govern the manipulation of symbols in expressions. As students become more familiar with these types of abstractions, they develop the means to solve equations and inequalities, express equivalent forms, and assert proofs.

Facility with abstraction and deeper knowledge of functions and relations give students more powerful mathematical tools to analyze and describe situations. Tools such as graphs and other visual representations of phenomena provide additional insights into problems and applications.

Standards in the Algebra I strand include performing operations such as opposite (additive inverse), reciprocal, and root; solving equations and inequalities with absolute values; simplifying expressions; solving multistep problems with linear equations and inequalities; graphing linear equations and finding the x - and y -intercepts; verifying points on a line given an equation; deriving linear equations; understanding and using the relationship between parallel lines and slopes; solving systems of linear equations, including meaningfully interpreting their graphical representations; performing operations and solving multistep problems with monomials and polynomials; and solving rate, work, and percent mixture problems.

The ten specific California academic content standards covered by the CAHSEE Algebra I strand are discussed in the following pages.

Strand	Algebra I (1A)	<div>If $x = -7$, then $-x =$</div> <div>A -7</div> <div>B $-\frac{1}{7}$</div> <div>C $\frac{1}{7}$</div> <div>D 7</div> <div>M02863</div>
Standard	1A2.0	
Students understand and use such operations as taking the opposite, finding the reciprocal, <u>and</u> taking a root, and raising to a fractional power . They understand and use the rules of exponents.*		
1 test question		

Familiarity with inverse operations is critical for students as they solve equations and inequalities. CAHSEE questions for this content standard focus on two of the stated components: finding the opposite (additive inverse) and finding the reciprocal. As students gain competency in algebraic reasoning, they should recognize the usefulness of finding the additive inverse and reciprocal in simplifying equations and inequalities.

Sample Test Question

The correct answer is choice D. Students should change the sign of -7 to 7 .

Analysis of Distractors

Distractor A: did not change the sign

Distractor B: took the reciprocal

Distractor C: took the reciprocal and the opposite

D: correct answer

* The CAHSEE test blueprint does not include the crossed-out portion of this content standard.

Strand	Algebra I (1A)	If x is an integer, what is the solution to $x - 3 < 1$?
Standard	1A3.0	
Students solve equations and inequalities involving absolute values.		
1 test question		
		<p>A $\{-3\}$</p> <p>B $\{-3, -2, -1, 0, 1\}$</p> <p>C $\{3\}$</p> <p>D $\{-1, 0, 1, 2, 3\}$</p> <p style="text-align: right;">M03035</p>

For success in algebra, students should understand the concept of absolute value and the ways its meaning is used in solving equations and inequalities. CAHSEE questions for this standard focus on both of its components: solving equations involving absolute values and solving inequalities involving absolute values. On the examination, the tested inequalities will involve only integers.

Sample Test Question

The correct answer is choice C. Students should recognize that this inequality, $|x - 3| < 1$, is equivalent to $-1 < x - 3 < 1$. By adding 3 to each member of the inequality, the following equivalence is obtained: $2 < x < 4$. Since x is an integer, the solution is 3 because 3 is the only integer greater than 2 and less than 4.

Analysis of Distractors

Distractor A: incorrect sign; misunderstood absolute value

Distractor B: set up the inequality improperly and then added -3 to only two members of the inequality

C: correct answer

Distractor D: set up the inequality improperly and then added -3 to only two members of the inequality

Strand	Algebra I (1A)	<p>Which of the following is equivalent to $4(x + 5) - 3(x + 2) = 14$?</p> <p>A $4x + 20 - 3x - 6 = 14$</p> <p>B $4x + 5 - 3x + 6 = 14$</p> <p>C $4x + 5 - 3x + 2 = 14$</p> <p>D $4x + 20 - 3x - 2 = 14$</p> <p style="text-align: right;">M02936</p>
Standard	1A4.0	

Students simplify expressions before solving linear equations and inequalities in one variable, such as $3(2x - 5) + 4(x - 2) = 12$.

2 test questions

Finding ways to simplify expressions before trying to solve an equation or inequality is a valuable problem-solving skill. CAHSEE questions for this content standard involve both equations and inequalities and focus on simplification rather than solution.

Sample Test Question

The correct answer is choice A. Students should understand that, for the given equation, the first step in simplifying is to expand the quantities in parentheses with each coefficient:

$$4x + 20 - 3x - 6 = 14.$$

Analysis of Distractors

A: correct answer

Distractor B: did not multiply 4 by 5 in the first set of parentheses and obtained a product of +6 when multiplying -3 and 2

Distractor C: did not multiply 4 by 5 in the first set of parentheses and did not multiply -3 by 2 in the second set of parentheses

Distractor D: did not multiply -3 by 2 in the second set of parentheses

Strand	Algebra I (1A)	Solve for x.
Standard	1A5.0	$5(2x - 3) - 6x < 9$
Students solve multistep problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.		A $x < -1.5$
		B $x < 1.5$
		C $x < 3$
		D $x < 6$
1 test question		M02938

Solving multistep problems and word problems can help students develop problem-solving skills and enhance their ability to think algebraically. CAHSEE questions for this standard focus on both of its components: finding solutions to linear equations and inequalities and providing justification for each step in the solution. Justification requires the identification of appropriate specific steps in the solution process that contribute to solving the equation or inequality.

Sample Test Question

The correct answer is choice D. Students should recognize that the inequality requires expanding the quantity in parentheses by multiplying it by the coefficient, combining like terms, adding 15 to both sides of the equation, and dividing both sides by 4 to obtain the value for x .

Analysis of Distractors

Distractor A: subtracted 15 in the third step

Distractor B: subtracted 15 in the third step and divided by -4 in the fourth step, or found $+6$ as the sum of 15 and 9

Distractor C: found $8x$ as the sum of $2x$ and $-6x$ after not multiplying 5 and 2 in the first set of parentheses

D: correct answer

Strand Algebra I (1A)

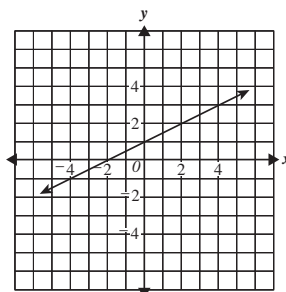
Standard 1A6.0

Students graph a linear equation and compute the x - and y -intercepts (e.g., graph $2x + 6y = 4$). They are also able to sketch the region defined by linear inequality (e.g., they sketch the region defined by $2x + 6y < 4$).*

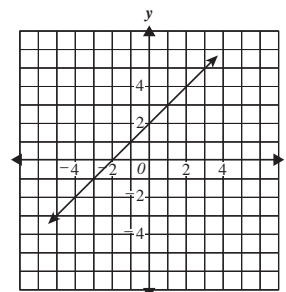
2 test questions
(1 graphing and
1 computing)

Which of the following is the graph of $y = \frac{1}{2}x + 2$?

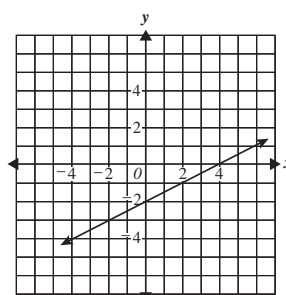
A



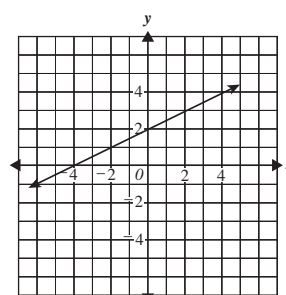
C



B



D



M02026

CAHSEE questions for this standard focus on two components: graphing the linear equation and finding the x - and y -intercepts. Students may be asked to identify the graph that corresponds to a given equation or to identify the equation that corresponds to a given graph. The x - and y -intercepts may be identified by a single number or a coordinate pair.

Sample Test Question

The correct answer is choice D. Students should recognize that the equation, $y = \frac{1}{2}x + 2$, is in the form $y = mx + b$ and use this information to determine the correct graph: The variable b represents the y -intercept, which in this case is 2, and the variable m represents the slope of the graph, which in this case is $\frac{1}{2}$.

* The CAHSEE test blueprint does not include the crossed-out portion of this content standard.

Analysis of Distractors

Distractor A: a line with a slope of $\frac{1}{2}$ and y-intercept of 1

Distractor B: a line with a slope of $\frac{1}{2}$ and y-intercept of -2

Distractor C: a line with a slope of 1 and y-intercept of 2

D: correct answer

Strand	Algebra I (1A)	Which of the following points lies on the line $4x + 5y = 20$?
Standard	1A7.0	
Students verify that a point lies on a line, given an equation of the line. Students are able to derive linear equations. by using the point slope formula.*		A (0, 4)
		B (0, 5)
		C (4, 5)
		D (5, 4)
1 test question		M02565

CAHSEE questions for this standard require students to select a set of one or more points, either by ordered pairs or by graphical location, that lie on the graph of a given linear equation, or to select an equation, either by its algebraic notation or by its graph, whose graph includes one or more specified points. Components of the standard include verifying that a point lies on a given line and deriving an equation from information given about the line. To verify that points do or do not lie on a given line, students may use substitution of x - or y -values to find corresponding ordered pairs.

Sample Test Question

The correct answer is choice A. Students should recognize that the equation $4x + 5y = 20$ represents true statements for certain corresponding pairs of values for x and y . The values may be tested by substituting them for x and y , respectively, in the equation. If $x = 0$ and $y = 4$, then $4(0) + 5(4) = 20$ is a true statement.

Analysis of Distractors

A: correct answer

Distractor B: substituted 0 for x and 5 for y , the result being that $4(0) + 5(5) = 25$ rather than 20

Distractor C: substituted 4 for x and 5 for y , the result being that $4(4) + 5(5) = 41$ rather than 20

Distractor D: substituted 5 for x and 4 for y , the result being that $4(5) + 5(4) = 40$ rather than 20

* The CAHSEE test blueprint does not include the crossed-out portion of this content standard.

Strand	Algebra I (1A)	What is the slope of a line parallel to the line
Standard	1A8.0	$y = \frac{1}{3}x + 2$?
Students understand the concepts of parallel lines and perpendicular lines and how their slopes are related.		A -3
Students are able to find the equation of a line perpendicular to a given line that passes through a given point.*		B $-\frac{1}{3}$
1 test question		C $\frac{1}{3}$
		D 2
		M02653

To demonstrate understanding of this content standard, students must know that parallel lines have equivalent slopes and different x - and y -intercepts. CAHSEE questions for this standard may require students to find the slope of a line parallel to a given line, to identify pairs of parallel lines from their slopes, or to identify lines not parallel to a given line from a given or derived slope.

Sample Test Question

The correct answer is choice C. Students should know that parallel lines have equivalent slopes. They must also recognize that the slope of the line that is represented by the equation

$$y = \frac{1}{3}x + 2 \text{ is } \frac{1}{3}.$$

Analysis of Distractors

Distractor A: the negative reciprocal of the slope of the given equation

Distractor B: the additive inverse of the slope of the given equation

C: correct answer

Distractor D: the y -intercept of the given equation

* The CAHSEE test blueprint does not include the crossed-out portion of this content standard.

Strand	Algebra I (1A)	$\begin{cases} 7x + 3y = -8 \\ -4x - y = 6 \end{cases}$ <p>What is the solution to the system of equations shown above?</p> <p>A $(-2, -2)$</p> <p>B $(-2, 2)$</p> <p>C $(2, -2)$</p> <p>D $(2, 2)$</p> <p>M02956</p>
Standard	1A9.0	
Students solve a system of two linear equations in two variables algebraically and are able to interpret the answer graphically. Students are able to solve a system of two linear inequalities in two variables and to sketch the solution sets.		
1 test question		

Many real-world situations are most appropriately modeled as systems of equations, and graphs of these kinds of systems are common in newspapers and other media. CAHSEE questions for this content standard focus on four components: solving systems of linear equations, interpreting the solutions graphically, solving a system of linear inequalities, and determining the solution sets.

Sample Test Question

The correct answer is choice B. To solve this sample problem, students should use a method such as the following:

(multiply the second equation by 3)	$-12x - 3y = 18$
(add to the first equation)	$-5x = 10$
(divide by 5)	$x = -2$
(substitute $x = -2$ in the first equation)	$7(-2) + 3y = -8$
(expand)	$-14 + 3y = -8$
(combine)	$3y = 6$
(divide by 3)	$y = 2$

Analysis of Distractors

Distractor A: error in substitution and/or computation

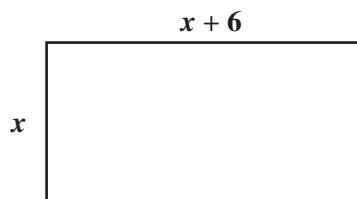
B: correct answer

Distractor C: misunderstood the correct order of the x - and y -values

Distractor D: error in substitution and/or computation

Strand **Algebra I (1A)**Standard **1A10.0**

Students add, subtract, multiply, and divide monomials and polynomials. Students solve multistep problems, including word problems, by using these techniques.

1 test question

The length of the rectangle above is 6 units longer than the width. Which expression could be used to represent the area of the rectangle?

- A** $x^2 + 6x$
- B** $x^2 - 36$
- C** $x^2 + 6x + 6$
- D** $x^2 + 12x + 36$

M00402

This standard requires students to accurately execute arithmetic operations on monomials and polynomials and to select and use these techniques to solve problems.

Sample Test Question

The correct answer is choice A. Students must know to multiply the length by the width in order to find the area and then perform the computation correctly.

Analysis of Distractors

Distractors B, C, and D represent examples of incorrect multiplication of the monomial by the binomial.

Strand	Algebra I (1A)	Mr. Jacobs can correct 150 quizzes in 50 minutes. His student aide can correct 150 quizzes in 75 minutes. Working together, how many minutes will it take them to correct 150 quizzes? A 30 B 60 C 63 D 125
Standard	1A15.0	
Students apply algebraic techniques to solve rate problems, work problems, and percent mixture problems.		
1 test question		
		M03000

The key assessment of student learning in any subject is whether or not students can apply their knowledge and skills to a new problem situation. CAHSEE questions written for this standard focus on students' ability to apply their mathematical skills and knowledge to solve rate problems, work problems, and percent mixture problems.

Sample Test Question

The correct answer is choice A. Students should recognize that correcting 150 quizzes in 50 minutes is equivalent to the rate of 3 quizzes per minute and that correcting 150 quizzes in 75 minutes is equivalent to the rate of 2 quizzes per minute. Working together, Mr. Jacobs and his aide can correct 5 quizzes each minute: $150 \text{ quizzes} \div 5 \text{ quizzes/minute} = 30 \text{ minutes}$.

Analysis of Distractors

A: correct answer

Distractor B: the number of minutes it would take both individuals to complete the task at the rate of 2.5 quizzes/minute—their average rate

Distractor C: the approximate average of their time for 150 quizzes

Distractor D: the sum of the number of minutes each takes to correct 150 quizzes ($50 + 75$)



Appendices

Appendix A

Test and Item Development

The questions that appear on the CAHSEE have been through an extensive development process to ensure that they are valid and fair measures of what students know and are able to do.

Content Validity

To ensure that the CAHSEE is a valid measure of the specified academic content standards, the questions are carefully designed to assess the content indicated in the test blueprints. Insofar as possible, each question requires students to demonstrate knowledge and/or skills in only one standard. Because many academic content standards cover a wide range of knowledge and skills, individual questions may assess one component of the standard. Other questions may address underlying, foundational knowledge or skills required for higher achievement in the standard.

Technical Quality

Well-written questions give students an opportunity to demonstrate what they know and are able to do; students do not have to guess what the question is asking. When questions are clearly written and easily understood, students are able to provide evidence of their learning. Questions have only one clearly correct answer. The language is simple, direct, and free of ambiguity. Questions should not test reading ability or vocabulary if that is not the purpose. CAHSEE questions are reviewed for content validity and technical quality by committees of California educators.

Test Bias

Bias in testing can take several forms, including the use of unfamiliar or insensitive language and terms, the presentation of stereotypes, and the inclusion of concepts that are offensive or negative toward any group. During the development process, CAHSEE questions are continually reviewed for potential bias to ensure that the CAHSEE meets the highest professional testing standards.

For general matters of style and grammar, CAHSEE test developers consult *The Gregg Reference Manual*, *Merriam-Webster's Collegiate Dictionary*, and *The American Heritage College Dictionary*.

The following checklists are used by CAHSEE item writers and review committees as a basis for evaluating the content validity, technical quality, and fairness of questions. Teachers may use these checklists to improve classroom assessments. For multiple-choice questions for mathematics, teachers may wish to write standards-based test questions to help students prepare for the CAHSEE. The checklist in Table 1 is provided for teachers to evaluate their own questions against the general requirements for CAHSEE multiple-choice questions.

Table 1
Development Checklist for Multiple-Choice Questions

✓	Requirements
The item as a whole—	
	measures the objective (content standard).
	follows the test specifications.
	uses grade-appropriate vocabulary and sentence structures.
	reflects current teaching practices.
	is free of bias, sensitive language or topics, and stereotypes.
	has a clear purpose.
	tests worthwhile (not trivial) concepts or information.
	is grammatically correct.
	is factually accurate.
	clearly presents one central idea.
	has one clearly correct answer.
	contains simple, direct, unambiguous language.
	is within the appropriate range of difficulty.
	follows appropriate style guidelines.
	does <u>not</u> ask for the student's opinion.
	does <u>not</u> use vocabulary and idiomatic phrases that could be unfamiliar.
	does <u>not</u> rely on students possessing outside knowledge.
	is <u>not</u> tricky or cute.

Development Checklist for Multiple-Choice Questions, continued

✓	Requirements
The stem of the item—	
	gives the test taker a full sense of what the item is asking.
	is clear and concise.
	is either a question or an incomplete statement.
The stimulus or passage for the item—	
	provides all the information needed to answer the items.
	is correctly and clearly labeled.
	is required to answer the associated item(s).
	is likely to be interesting to students.
The response options—	
	are written so that no one option is significantly different from the others.
	relate to the stem in the same way.
	include plausible and reasonable misconceptions and errors.
	have a balance of A, B, C, and D responses.
	do <u>not</u> contain an option that denies the truth of any other option.
	do <u>not</u> deny the truth of the stem.
	do <u>not</u> give clues to students, such as the use of absolutes or repeating key words that appear in the stem.
	do <u>not</u> repeat words that could be placed in the stem.
	do <u>not</u> include distractors that are phrased differently but have the same meaning.

Test Security

One of the most significant guarantors of fairness to all students who take the CAHSEE is that passages, writing prompts, graphical materials, and questions remain secure at all times. Individuals who circumvent or attempt to circumvent procedures to maintain test security diminish the legitimate and honest efforts of all other students and teachers to participate in the state's assessment system. The California Department of Education (CDE) has the authority, according to the California *Education Code* Section 60851 (b) and (c) and the copyright statutes of the United States, to act against any individual or group of individuals who knowingly attempt to copy, duplicate, or transmit in any way, the contents of secure material from test booklets, answer documents, in whole or in part, to any other individual or group of individuals. The California Department of Education may employ procedures to maintain the test security of the CAHSEE, including but not limited to monitoring of test administration, document handling, and post-test analytic techniques such as mark discrimination analysis.

Appendix B

Resources

Student Study Guides

CDE has released a Study Guide for the mathematics portion of the CAHSEE. The Study Guide features answers to frequently asked questions, test-taking tips, and a practice test. The Study Guide includes released test questions with the solutions explained. While the Study Guide was written for students to use independently, teachers may incorporate it into their classroom instruction to prepare students for the CAHSEE. Additionally, the Study Guide is available on the CDE Web site.

Resource Documents

The information in this Teacher Guide is based on the California academic content standards and the California frameworks in mathematics. These documents may be ordered from the California Department of Education, or they may be downloaded from the CDE Web site, as shown below:

The Mathematics Content Standards for California Public Schools, Kindergarten Through Grade Twelve (1997) is available from the California Department of Education, CDE Press, Sales Unit, 1430 N Street, Suite 3207, Sacramento, CA 95814; 1-800-995-4099, ext. 1. It is also available at <http://www.cde.ca.gov/> on the Internet.

The Mathematics Framework for California Public Schools, Kindergarten Through Grade Twelve (1999), is available from the California Department of Education, CDE Press, Sales Unit, 1430 N Street, Suite 3207, Sacramento, CA 95814; 1-800-995-4099, ext. 1. It is also available at <http://www.cde.ca.gov/> on the Internet.

Other Resources

The student Study Guides for the CAHSEE are available at <http://www.cde.ca.gov/ta/tg/hs/resources.asp>

Released Test Questions from the CAHSEE are available at <http://www.cde.ca.gov/ta/tg/hs/resources.asp>

Answers to Frequently Asked Questions (FAQs) are available at <http://www.cde.ca.gov/ta/tg/hs/faq.asp>

Blueprints for the CAHSEE are available at <http://www.cde.ca.gov/ta/tg/hs/resources.asp>

CAHSEE background information is available at <http://www.cde.ca.gov/ta/tg/hs/>

Appendix C

Glossary of Terms Used in This Guide

Answer Choices — The correct answer and the distractors in a multiple-choice test question.

Blueprint — The plan for assessment that specifies the number of questions on each test form according to strand and academic content standard.

Clueing — An instance in which one test question provides information that could be used to select the correct answer to another question, or an instance in which the stem in a multiple-choice question clues the correct answer.

Distractors — Incorrect answers to a multiple-choice stem.

Field-Test Questions — Test questions that are administered to students to gain information about the quality of the question. Student performance on these questions does not affect student scores.

Foundational Knowledge/Foundational Skill — Knowledge or skill that a student would be taught and be expected to know prior to taking courses covering the academic content standards tested in the CAHSEE.

Item — A test question written in one of several possible item formats.

Item Format — The basic design of a test question (e.g., multiple-choice, constructed response).

Key — The correct answer to a multiple-choice question.

Multiple-Choice Question — A stem plus a number of response options or answer choices (four for CAHSEE).

Response Options — The choices in a multiple-choice question, consisting of one key (correct answer) and a number of distractors (three for CAHSEE).

Scoring Guide — The rubric or protocol to follow when assigning a point value to responses to a writing task.

Specifications — The document that includes a description of how each standard is assessed on the CAHSEE.

Standard — A statement of what students should know and be able to do.

Stem — The initial part of a multiple-choice test question in which the task or premise is given. The stem may be a question, an incomplete statement, or a set of directions.

Stimulus — A picture, graph, map, chart, quotation, or other text that students are asked to interpret when answering a test item.

Strand — A category of standards that relate to each other for purposes of reporting performance on the CAHSEE.

Mathematics Teacher Guide

July 2008